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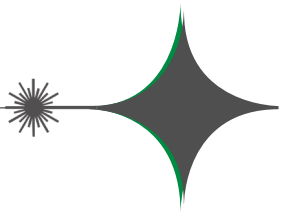


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# Study of the Efficacy of Irradiated Fungus Beauveria bassiana to Control Fig Moth Pupae.

Hamid Kadhum Saoud [1]

[1] Karbala Education Directorate

*P A P E R I N F O*

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*A B S T R A C T*

The ability of the fungus *Beauveria bassiana* was tested when exposed to ultraviolet light at a wavelength of 312nm for 4 and *8minutes.*A significant decrease was observed in the rates of adult emergence from fig moth pupae *Ephestia cautella* treated with fungi which was exposed to radiation with increasing fungal concentrations. Additionally, the percentage of albumin in the blood was decreased by 14.30 and 16.10µg.ml$^{-1}$ for adults of the fig moth, that emerged from pupae at the age of (1-2) and (6-8)days, respectively, when sprayed to fungus solution irradiated for 4minutes at a concentration of 1.5×10$^{7}$spore.ml$^{-1.}$ . This result was compared with adult insects resulted from pupae, which sprayed to a fungus solution that was not exposed to radiation. They reached 28.12 and 28.15µg.ml$^{-1}$, separately, and the amount of glucose in the blood also decreased, reaching 24.00 and 34.00µg.ml$^{-1}$ for adult insects that resulted from pupae at the age of (1-2) and (6-8)days, respectively. The number of the hemocytes decreased 119 and 155cells, individually, for adult insects that resulted from pupae sprayed to the irradiated fungus, while 185 and 220cells to insects that resulted from pupae sprayed with the same concentration of non-irradiated fungal solution.

## 1. INTRODUCTION

Insects are one of the main causes of economic and qualitative losses that occur in stored and packaged foodstuffs, including dates, as they cause great economic damage.This destruction takes place as a result of the spoilage of those dates. The insect *Ephestia cautella* is one of the most common insects that infect dates, and the saw-grain beetle *Oryzaephilus surinamensis*(L.) also infects dates. Thus, insect infestation remains a constant and serious threat. Dates are regarded as one of the best food crops in Iraq, but stored dates are infested with a large number of insects, some of which belong to the genus *Ephestia* spp. of the order Lepidoptera. It was mentioned[1] that the larvae of the fig moth infect the dates falling from the palm tree and the dates are left after harvesting.More over the infection starts from the orchard and continues in stores throughout the months of the year markets and consumption. The types of date moths, which are *E. cautella*, *E. calidella* and *E. figulilella*, are among the species that spread globally.They are found to attack dates, raisins, figs, field pistachios and dried grapes, causing economic losses if they are not controlled in the field and in the store[2]. Two main factors make this pest danger to stored dates.These are the ability of the insect to develop resistance against chemical insecticides as well as the tendency of the larvae of this species to feed inside the date fruit. It was stated [3] that the genus Ephestia is one of the most important pests in flour factories in hot countries, as it causes severe damage to all stored winter products. Inaddition, the infection rates may reach about 42% if it is not effectively controlled. Thus, recent studies focused on this field, including searching for modern means to control insect pests in the Iraqi environment. Chemical control is one of the traditional methods of controlling insects that have shown a high ability to kill pests. However, the long-term negative effects of the use of pesticides and what they cause of pollution to the environment , the risks to human health and domestic animals and finally the imbalance they cause in the natural balance through their negative impact on vital enemies are the interests of research.The current study investigates using biological resistance elements in order to protect the crop without harming the environment. Among the most important elements that

* Corresponding Author Institutional Email:
ewwtfdg@gmail.com (Hamid Kadhum Saoud)

have proven efficient and received wide attention are fungi that are pathogenic to insects. The pathogenic fungus *B. bassiana* is utilized by contaminating males of the red palm weevil, *R. ferrugineus*, with the fungus and releasing it into palm groves. Males are released[4], and among the physical methods that are commonly used during the last decades of the last century and the current one in controlling store insects is the use of electromagnetic energy such as infrared, gamma and ultraviolet rays[5].

## 2. MATERIALS AND METHODS

### 2.1. Collecting and Breeding Insects

For the purpose of conducting experiments, a laboratory colony of *E. cautella* was used, which was raised on an artificial nutrient media (81% wheat groats, 12% glycerine, 6% date molasses, and 1%yeast) according to the method mentioned in[6]. 125g of industrial food was placed inside a sterile glass bottle with a diameter of 14cm and a height of 22cm, then 10 pairs of newly emerged insects were released into it. 25±2C°, relative humidity 50 ±5%, and lighting duration (8L:16D) hours for 25 days, which were monitored daily for the purpose of obtaining pupae up to adult insects. The larvae appeared and developed into the fifth larval stage, as they were observed in a wandering state for the purpose of preparation for pupation. Breeding lasted for two generations before being tested. Five replications were used for each concentration of fungus, and in each replication five pupae for each age group . The death rate was calculated from Equation (1) given in[7].

| | |
|---|---|
| % killing =(treatment in killed insects No.)/(total insects No.)x100% | (1) |

Corrected death rates were also calculated from Equation (2) given in[8].

| | |
|---|---|
| %Corrected kill = (%kill in control - % kill in experiment) / (% kill in control -100)X100% | (2) |

### 2.2. Preparation of the Main Concentrate of the Fungus

Use a container plate on the pure fungus colony to prepare the basic spore suspension (colony age 12days). After that, 5ml sterile distilled water containing Tween-20 solution as a moisture preservative was added to it .Then, the spores were mixed well.The contents have been filtered using a funnel with a small piece of medical gauze.After that, it was placed in a 50 ml glass beaker, after which 5ml from distilled water was added to remove all spores. It was shaken for 4minutes with a vortex device in order to separate the spores from the mycelium. After obtaining the filtrate, distilled water was added to complete the volume to 100ml, which represented the basic suspension.[9] Then, a spores counting slide was used by taking 0.1ml of the base suspension and placing it on a counting slide at 40× magnification. The number of spores inside the number of cells reached 60 spores. We multiplied it by the conversion number $2.5x10^5$ using the equation: $60x\ 2.5x10^5 = 1.5x10^7$ spore.ml$^{-1}$ concentration of the basic solution[10]. Accordingly, it is considered a source for obtaining other concentrations.

### 2.3. Preparation of Different Concentrations of Fungus

A quantity of the basic fungus suspension was diluted to reach the required concentration for the fungus *B. bassiana* through the following equation: The quantity taken from the original fungal solution(ml)= The concentration of the fungicide solution required for the purpose of the experiment/ the concentration of the basic fungus solution.

For 30ml of fungal solution with a concentration of $10^6$ spore.ml$^{-1}$ of the basic solution, the relation: $10^6/10^7$ =0.1ml is used .Then the product of the relation, multiply by the amount of fungicide solution required, which is (30ml).Thus, the result becomes 3ml. This amount of the basic solution is taken and 27ml of distilled water is added to it. In this way, 30ml is obtained from the required concentration, which is $10^6$ spores.ml$^{-1}$. In the same way, concentrations are obtained. The other ($1.5\times10^4$,$10^5$,$10^6$ and $10^7$spores.ml$^{-1}$) flasks are kept in the refrigerator at 4C$^0$.

### 2.4. To Obtain Pupae from Inside the Breeding Bottles

They were collected carefully with forceps, and divided into two groups, age (1-2) and (6-8)days. The pupae were placed in plastic dishes with a diameter of 10cm. It contains a filter paper that is moistened with distilled water, the conical flasks containing the concentrations of the fungal solution were exposed to UV rays for 4 and 8minutes from a distance of 20cm. After that, the pupae were sprayed in each repeater and for each age group with fungus. Regarding the control group, it was sprayed with a pathogenic fungal solution that was not exposed to radiation, and all samples were placed in volumetric flasks containing 50g of artificial food. All these samples were kept in the incubator at room temperature of 25 ±2C° and a humidity of 60± 5% for follow-up purposes. The results were analyzed by factorial method and using a completely randomized design (CRD), using the statistical analysis program GenStat2012.The differences between all samples were compared by using the L.S.D. At level of ($P<0.05$)[8].

## 3. RESULTS AND DISSCUTION

The results are shown in table 1and 2 the percentage of mortality of pupae increased, and consequently a decrease in the number of emerging pupae is noticed. The higher the concentration of the insect-pathogenic fungus *B. bassiana* exposed to ultraviolet radiation, the incidence of deformities increased in adult insects which resulted from pupae at the age of (1-2) and (6-8) days. They were sprayed to insect-pathogenic fungus exposed to ultraviolet radiation for periods of 4 and 8 minutes. It was also noted that the resulting insect movement was weak due to the emergence of fungus filaments. It was characterized by lethargy, and with the passage of time all parts of the insect's body became covered with fungal hyphae, and the hyphae of *B. bassiana* were white and smooth and grew externally after placing infected insects on the filter paper is moistened with distilled water. Also, death in newly spawned insects can be attributed to mycotoxins produced by fungal solution, experiments have shown that entomopathogenic fungi produce toxic substances that negatively affect the metabolic processes of insects[11]. Insects infected with the pathogenic fungus may have died because of the toxins of that fungus and not because of the fungus growing on those infected insects. In addition, the decrease in the numbers of immune cells of infected insects leads to a weakening of the immune system of emerging insects, and this is indicated by[12] when he studied the effect by fungal solution *B. bassiana* on the larvae of the lesser cotton leaf worm *S. exigua*.

An increase in the killing rates in pupae was observed with the increase in the fungal solution concentration, as the highest rates of mortality were 72.30 and 62.33% by concentrate at $1.5x10^7$ spores.$ml^{-1}$ for the pupae at the age of (1-2)days.More over, the irradiation period of the fungicide solution was 4 and 8minutes, respectively. As for the killing rates of pupae at the age of (6-8)days and with the same concentration and time periods, they showed a clear decrease, as they were 44.30 and 42.50%, respectively(Tables1 and2).This is due to the secretion of toxins from fungi that are pathogenic to insects and the growth of its filaments. As it is explained in [13] that the fungus tissues grow and replicate inside the body of the infected insects after penetration, and the development of the fungus within the blood lymph of the insect lead to a decline in the functioning of the immune system .Therefore, the fungal filaments penetrate all the internal tissues, after which they penetrate the body wall and exit on the outer surface.This leads to difficulty in movement, [14] stated that the effect of mycotoxins extended to insects produced from pupae that were sprayed to the fungus. The results also showed a clear decrease in the levels of blood glucose since the percentages of fungus concentrations increased. The decrease was more in pupae at (1-2)days while pupae at (6-8)days, as it reached 24.00 and 34.00µg.$ml^{-1}$ in insects that emerged from pupae treated with age (1-2) and (6-8)days, respectively(Table1), treated with fungus at a concentration of $1.5\times10^7$spores.$ml^{-1}$ for 4minutes, compared to 10.20 and 23.80 µg.$ml^{-1}$, respectively (Table2) for insects emerging from pupae. Treatment with the same concentration and the same age for a period of 8minutes also, albumin levels decreased in the blood of insects which emerging from pupae at the same concentrate and age, as it decreased significantly to 14.30 and 10.50 µg.$ml^{-1}$ for pupae aged(1-2) days at a concentration of $1.5\times10^7$spore.$ml^{-1}$. The duration of exposure was 4 and 8minutes, separately, while to 12.20 and 16.10µg.$ml^{-1}$, exposure time of 4 and 8 minutes, respectively, for insects that emerged from pupae at (6-8)days and the same concentrate. As for the number the hemocytes in insects that resulted from pupae, they decreased clearly with the increase of the fungal solution. The number of cells was 119 and 155 cells.$ml^{-1}$ for adults that emerged from pupae treated with a fungal solution of $1.5\times10^7$ spores.$ml^{-1}$ at ages (1-2) and (6-8)days for 4 minutes(Table1). The number of cells was 70 and 86 cells/ml for adults that results from pupae that was treated by the same fungal concentrate itself for 8 minutes (Table2). [15]indicated that ultraviolet radiation causes changes in the composition of the genetic material inside the nucleus.Thus, the occurrence of various genetic mutations has a significant impact on the composition and vitality of the pathogenic fungus. [16] It was mentioned that the females of the desert locust insect, when treated with fungus exposed to ultraviolet radiation, their ovaries became smaller, reaching 50mm while 82mm in unexposed individuals, and there was also an extraction in the tissues of the digestive system of locust nymphs treated with fungus, compared to the control group. Through the results, it was found that the killing level did not reach 100%, except for $1.5\ x10^7$ spores.$ml^{-1}$ exposed to radiation for a period of 4 minutes reached 72.30% for pupae aged (1-2)days after 12days (Table1). The insect's immune system may be able to defend whenever the solutions are low, and the system's effectiveness decreases whenever the solutions are high in density, and this is what he mentioned[17] as his results about effectiveness of the *B. bassiana* fungus on activity of the *P. operculella* insect explained the immune system the insect can only defend low densities. The tables1 and2 clearer that the pupae of age of (1-2)days were more affected by the fungus solutions than the pupae at the age of (6-8)days, as the statistical analyzes which used the clear differences between the percentage of death and a decrease in the percentages of glucose and albumin and the amount of blood cells in the blood, one of the possible reasons is that the cuticle of virgins at (1-2) days is less thick and more flexibile than those of pupae at the age of (6-8) days.Accordingly, it becomes easier to penetrate by fungal hyphae compared to other older ones.The reason behind that may be the different formations of the body wall insect, such as the presence of a waxy layer.In addition , there are several factors that depend on the type of insect and the insect's environment like humidity that help in germination of fungal spores.

**Table1.** Demonstrates effect of fungus treated with ultraviolet light for 4minutes on pupae at the age of (1-2) and (6-8) days and its development of E. cautella.

| (6-8) days | | | | (1-2) days | | | | fungus concentrate |
|---|---|---|---|---|---|---|---|---|
| No. hemocyte /ml | The glucose µg.ml$^{-1}$ | The albumin µg.ml$^{-1}$ | The death rate % | No. hemocyte /ml | The glucose µg. ml$^{-1}$ | The Albumin µg. ml$^{-1}$ | The death rate % | spore .ml$^{-1}$ |
| 220 | 55.10 | 28.15 | 26.00 | 185 | 42.05 | 28.12 | 42.10 | Cont. ($10^6$ ×1.5) |
| 202 | 52.00 | 27.00 | 25.02 | 180 | 40.20 | 1225. | 45.05 | $10^4$ × 1.5 |
| 180 | 45.10 | 23.30 | 39.33 | 162 | 36.15 | 22.10 | 52.50 | $10^5$ × 1.5 |
| 164 | 39.10 | 19.10 | 42.10 | 143 | 30.01 | 20.00 | 68.20 | $10^6$ × 1.5 |
| 155 | 34.00 | 1016. | 44.30 | 119 | 24.00 | 14.30 | 72.30 | $10^7$ × 1.5 |
| 0.48 | 0.85 | 0.88 | 9.00 | 8.20 | 11.60 | 5.19 | 5.98 | L.S.D. |

**Table2.** Demonstrates effect of fungus exposed to ultraviolet light for 8minutes on pupae aged(1-2) and (6-8)days and its development to E. cautella.

| (6-8) days | | | | (1-2) days | | | | fungus concentrate |
|---|---|---|---|---|---|---|---|---|
| No. hemocyte /ml | The glucose µg. ml$^{-1}$ | The albumin µg. ml$^{-1}$ | The death rate % | No. hemocyte /ml | The glucose µg. ml$^{-1}$ | The albumin µg. ml$^{-1}$ | The death rate % | spore. ml$^{-1}$ |
| 242 | 42.00 | 32.90 | 33.30 | 208 | 40.10 | 16.15 | 44.00 | Cont. ($10^6$ ×1.5) |
| 210 | 34.50 | 2.102 | 38.20 | 198 | 36.20 | 18.30 | 48.30 | $10^4$ × 1.5 |
| 183 | 33.00 | 18.20 | 34.00 | 176 | 30.33 | 16.20 | 54.10 | $10^5$ × 1.5 |
| 125 | 28.00 | 14.50 | 38.66 | 115 | 28.15 | 12.20 | 58.00 | $10^6$ × 1.5 |
| 86 | 23.80 | 12.20 | 42.50 | 70 | 20.10 | 10.50 | 62.33 | $10^7$ × 1.5 |
| 3.69 | 5.60 | 0.51 | 12.476 | 6.41 | 1.03 | 12.10 | 13.34 | L.S.D. |

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## Arabic Abstract

تم اختبار قدرة الفطر *Beauveria bassiana* عند تعريضه للأشعة فوق البنفسجية عند طول موجي 312 نانومتر لمدة 4 و 8 دقائق. ولوحظ انخفاض معنوي في نسب بزوغ البالغات من عذارى عثة التين *Ephestia cautella* المعاملة بالفطر المعرض للإشعاع مع زيادة التراكيز الفطرية، وانخفضت نسبة الألبومين في الدم بمقدار 14.30 و16.10 ميكروغرام.مل$^{-1}$ في بالغات فراشة التين التي بزغت من العذارى بعمر (1-2) و(6-8)يوم على التوالي، عند رشها بمحلول فطري مشعع لمدة 4 دقائق بتركيز $1.5\times10^{7}$ بوغ.مل$^{-1}$ مقارنة بالحشرات البالغة الناتجة عن العذارى التي تم رشها بمحلول فطري غير معرض للإشعاع بلغت 28.12 و28.15 ميكروغرام.مل$^{-1}$ على التوالي. كما انخفض مستوى الجلوكوز في الدم ليصل إلى 24.00 و34.00 ميكروجرام.مل$^{-1}$ في الحشرات البالغة الناتجة عن العذارى بعمر (1-2) و(6-8) أيام على التوالي. انخفض عدد الخلايا الدموية 119 و 155 خلية على التوالي للحشرات البالغة الناتجة عن العذارى المرشوشة بالفطر المشعع، بينما انخفض عدد الخلايا الدموية 185 و 220 خلية للحشرات الناتجة عن العذارى المرشوشة بنفس التركيز من المحلول الفطري غير المشعع.

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# Diagnosis of Cochlicella Barbara in Baghdad Governorate

Fatima Abdulrahman Dohi [1] , Samia KH Mahmood [2]

[1] Middle Technical University , College of Medical and Health Techniques
[2] Al-Nahrain University, College of Biotechnology, Baghdad, Iraq

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*A B S T R A C T*

Environmental and laboratory studies have been completed to investigate the presence and spread of Banded Conical Snail, *Cochlicella barbara*: Linnaeus, in the Iraqi environment. Laboratory studies have included the detection and diagnosis of living microorganisms that cause respiratory diseases to humans and small ruminants during the period of 2018 – 2020.
The results of the detection indicated the presence of the *C. barbara* cone snail in Iraqi environment. The results of the international diagnosis also confirmed this type of snail. The results of the laboratory examination suggest that this snail is an intermediate host of some nematodes which infect humans and some animals; and cause respiratory diseases. The *Neostrongulus lineairs* was extracted from this snail, which was identified in international scientific centres.
The results of this research can suggest the authorities of the Ministry of Health and Ministry of Agriculture to take the necessary measures to reduce the spread of this disease, and to raise the scientific awareness for doctors and specialists when diagnosing the causes of certain respiratory diseases.

## 1. INTRODUCTION

Infectious diseases are a significant burden for many societies, including states' member of World Health Organization (WHO).
To reduce this burden, an integrated approach is needed that combines between health empowerment and disease prevention. The prerequisite for success in this fight is the participation of all health-care and other health workers in a position to contribute significantly in reducing the burden on health sectors [1].
There are many viruses and pathogens transmitted by legumes (insects and their like) and another group of invertebrates, including crustaceans, mosquitoes, ticks, flies, molluscs and snails. They cause a viral infection that mainly affects animals. But it can spread to humans, and it is transmitted to humans through these organisms by sting or oral transmission.Thus, it enters the human bloodstream and causes disease [3].

Corresponding Author Institutional Email:
fatima.abdulrahman@mtu.edu.iq (Fatima Abdulrahman Dohi )

Stated that the spread of cone snails, Banded Conical Snail, *Cochlicella barbara*: Linnaeus in Iraq is through cargoes from Australia, the home of this kind of snails, which attacks the sage crop and causes extreme effects [2].
Both [1] and [3] noted that the *C. barbara* cone snail is the host of some species of Ascaris that infect humans and animals and cause them disease.

## 2. WORKING METHODOLOGIES

1. Investigation of infestation: A survey of the presence of *C. Barbara* cone snails in south of Baghdad (Al-Mada'in) was conducted from October 2018 to September 2019 and samples were collected from home gardens.
2. Diagnosis: Samples of snails were placed in plastic tubes containing 90% alcohol. The samples were sent to the British Natural History Museum for accurate diagnosis. The samples were sent by Aramx International Rapid Mail.

## 3. RESULTS & DISCUSSION

Quantum Spread Investigation: Results indicate cochlear presence in home gardens, especially in thiel areas. Starting in December, and reaching the highest population density during February and March, this

snail is found at vegetation and bush during the night and descends at the surface of the soil by day. The cochlear climbs on some of the plants of the home gardens to feed, preferring to feed on fallen Naring fruits and being.

The purpose of this research is to draw attention to the seriousness of the *C. Barbara* cone snail as an intermediate family of certain nematodes that cause disease in humans and animals. This in turn serves sectoral stakeholders in the diagnosis and treatment of pathogens that is caused by this disease.

3.Isolation and diagnosis of nematodes living with snails: Five cropped cone snail were placed in a 9mm glass petri dish at its base. Drizzled water filtration paper containing 10 large wax worm larvae is a final development of *Galleria melonella* as a nematode trap. Two weeks later, the larvae were transferred to white trap to get the nematodes. The solution containing nematodes was placed in glass tubes and samples were sent for diagnosis at international scientific centres.

consumed with white pulp layer sparse (picture 1) [5] has stated that purebred snails are a pest on Australia's seeds crops, causing economic losses and moving with seeds through harvest vents because of their small size.

**Picture 1.** Nourishing on Cochlicella barbara fallen naring fruits

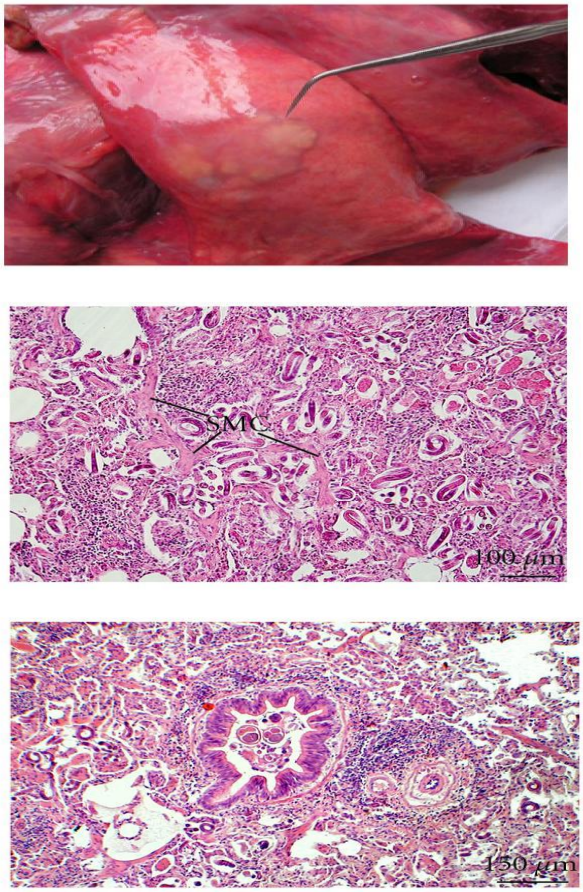
1. Diagnosis: The results of the diagnosis of samples which were sent to the British Natural History Museum showed that the cochlear is the cone snail: Banded Conical Snail *Cochlicella barbara*: Linnaeus, 1758 (Gastropoda: Cochlicellidae) (picture 2). The diagnosis was made by the Jon Ablett, a specialist under IAS 2019-3488, who mentioned that this is an alien to the Iraqi environment, native to Europe, and very widespread in Australia. It is considered a pest on the sage crop, causing serious damage. This confirms what [4] recorded the presence of this snail for the first time in the Iraqi environment in 2017.

Nematoda (filamentous worms) living with cone snails *C. Barbara*: The results of the diagnosis indicated that the nematode living with the *C. barbara* cone snail was the species of *Neostrongylus linearis* [4] (Nematoda, Protostrongylidae). [6] stated that they are common cylindrical worm's endemic throughout the tropical region (picture 4 and 5).However, they are also found worldwide in all climates. Infectious filamentous larvae can penetrate the skin, infect the human host and young

**Picture 2**. Embossed cone snail Cochlicella barbara.

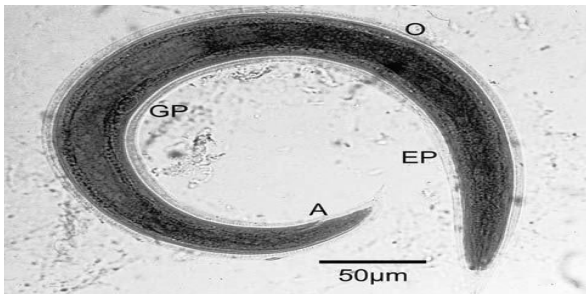
1- bodies and cause damage to the lungs (picture 3). The larvae migrate through soft tissue and enter the lungs through the bloodstream. The majority of the round worms above the bronchial tree migrate to the pelvis, swallow and enter the digestive system. The larvae can enter the circulatory system, return to the lungs and causing autoimmune infection. The entire life cycle of a heterosexual couple can be completed within a single parent [7] Control of cone snails *C. Barbara*

*2.* Kumar , [7] and (8) stated that Molluscicides are most effective in combating this snail when used with the attractive food graft containing frolic acid. This contributes to the lack of environmental pollution compared to the use of spraying pesticides alone. Iron phosphate, which is sold under many commercial names, can also be used as a safe use in terms of children, birds, fish, other wild animals, and domestic animals [9]. From the results of the research, the fruits of Naring can be used as an attractive taste with frolic acid in the fight against this snail.




**Picture 3.** Nematode infection of the lungs Neostrongulus linearis Sheep). Panayotova-Pencheva, M. S. and Alexandrov, M. T. (2010).

**Picture 4.** Neostrongulus linearis Living with cone snails Cochlicella Barbara




**Picture 5.** Phase III nematode larvae Neostrongulus linearis 0X, O (esophageal Oesohagus, GP (Genital primordium), EP (Excretory pore), A (Anus). Godan D. 1983

## 4. CONCLUSIONS

Research results showed the presence and spread of *C. barbara* cone snails in the home garden environment in Iraq. This snail is an intermediate provider of filamentous worms *Neostrongulus linearis*, which follows the family Protostrongylidae.The previous studies and research suggest that snail affects the human race as well as young bodies.It causes respiratory damage, especially to the lungs.

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**Competing interests:** Authors state no conflict of interest.

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## Arabic Abstract

تم الانتهاء من الدراسات البيئية والمختبرية للتحقق من وجود وانتشار الحلزون المخروطي النطاق *Cochlicella barbara: Linnaeus* في البيئة العراقية . شملت الدراسات المختبرية كشف وتشخيص الكائنات الحية الدقيقة المسببة لأمراض الجهاز التنفسي للإنسان والمجترات الصغيرة خلال الفترة 2018 – 2020

أشارت نتائج الكشف إلى وجود الحلزون المخروطي *C. barbara* في بيئة العراق . كما أكدت نتائج التشخيص الدولي وجود هذا النوع من القواقع. وتشير نتائج الفحص المختبري إلى أن هذا الحلزون يعد عائلاً وسيطاً لبعض الديدان الخيطية التي تصيب الإنسان وبعض الحيوانات؛ وتسبب أمراض الجهاز التنفسي . وتم استخراج ديدان *Neostrongulus linearis* من هذا الحلزون والذي تم التعرف عليه في المراكز العلمية العالمية.

ويمكن لنتائج هذا البحث أن تقترح على الجهات المختصة في وزارة الصحة ووزارة الزراعة اتخاذ الإجراءات اللازمة للحد من انتشار هذا المرض، ورفع الوعي العلمي للأطباء والمتخصصين عند تشخيص أسباب بعض أمراض الجهاز التنفسي.

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# A review: Relationship between Inhibin B level and some trace elements in female infertility

Rehab Jasim Mohammed [1], Inam Joudah Radhi [2], Zena T Omran [3*]

[1, 2] Department of chemistry, Collage of Education for Pure Science, University of Kerbala, Kerbala, Iraq.
[3] Department of Biology, Collage of Education for Pure Science, University of Kerbala, Kerbala, Iraq.

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The granulosa cells of the female ovary generate the transforming growth factor (TGF-) superfamily member, inhibin B. Instead of being a possible indicator of male and female poverty, it serves as a motivator for change. As part of the utero-placental unit's paracrine ovarian and testicular regulators, it is now known to have several paracrine functions. This study investigated the relationship between inhibin B levels and a few trace components in women who were infertile.

## 1. INTRODUCTION

The environment was seriously harmed by the war in Iraq, both before and after 2003, and infertility is a disorder of the reproductive system where a clinical pregnancy cannot be obtained after at least a year of unprotected sexual activity. Many chemical and radioactive materials were handled and destroyed by unskilled workers, resulting in numerous mishaps, deaths, and cases of cancer or infertility among the survivors. Very few studies have explicitly examined the relationship between conflict and infertility [1,2].
A significant number of factors, such as obesity rates, cardiovascular diseases, hormone-dependent tumors, developmental issues, chronic childhood illnesses, early puberty, altered gender ratios, altered maternal ages, infections of the reproductive system, diet, addictions, and stress, contribute to the fact that fifteen percent [15%] of childbearing couples experience infertility, which is widely recognized as a serious public health issue. It usually arises from toxic accumulation in the body as well as psychological or emotional stress [3,4,5]. Transforming growth factor- (TGF-) superfamily dimeric polypeptide hormones are called inhibitors. Inhibin B, a paracrine regulator of the ovarian and testicular systems, has several paracrine effects on the uteroplacental axis and is not a potential indicator of poverty for either sex [6,7,8].
Minerals that constitute less than 0.01% of an individual's total weight or that are required in doses between 1 and 100 mg daily are referred to as "trace elements" (TEs). Zinc (Zn), copper (Cu), chromium (Cr), cobalt (Co), selenium (Se), and iodine (I) are among the trace elements essential for human health. The human body also requires trace elements like molybdenum (Mo) and manganese (Mn). Proteins, transcription factors, and fundamental metabolic processes, including enzymatic reactions as components of complexes, all depend on trace elements for their proper operation [9,10].

For life to exist, inorganic trace elements such as iron (Fe), copper (Cu), zinc (Zn), and selenium (Se) must be consumed on a daily basis in very small amounts, usually less than 100 mg. Selenium, as a trace element, is essential to human health and biological function. In areas with significant soil deficiencies, supplementation is advised as part of public health policy, since growing evidence indicates that this element is essential for healthy animal and human growth and reproduction [11,12].

*Corresponding Author Institutional Email: ***Zina.t@uokerbala.edu.iq*** (Zena T Omran )

Severe hypozincemia increases the risk of infertility in both sexes, as zinc is essential for healthy spermatogenesis [13,14]. Copper (Cu) is an essential mineral found naturally in a wide variety of foods and dietary supplements. It serves as a cofactor for a number of enzymes involved in physiological processes, such as iron metabolism and energy generation [15].

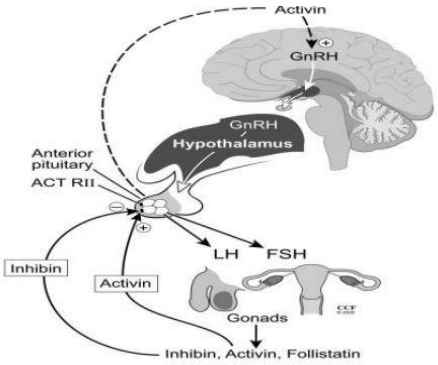
**Effect of inhibin B in female infertility**

Inhibin-B, on the other hand, shows an increase in the early follicular phase, followed by a drop, a brief peak immediately after the spike in luteinizing hormone (LH), and finally reduced values in the luteal phase. This pattern demonstrates that Inhibin-B, a granulosa cell product, participates in follicular growth and supports the hypothesis that serum concentrations reflect follicular function and oocyte quantity [16,17].

There are multiple controls over the production of inhibin B throughout life. The fact that inhibin B levels peak postpartum and are only weakly associated with a rise in serum follicle-stimulating hormone (FSH) suggests that Sertoli cell proliferation is most likely occurring during this developmental stage. Inhibin B levels decrease in response to FSH stimulation and remain low until adolescence when they rise again due to FSH's simultaneous control of spermatogenesis and growth [18,19,20].

Because inhibin B selectively decreases FSH, a high amount of this hormone may be the reason for the elevated LH/FSH ratio seen in certain individuals. Additionally, inhibin may directly promote androgen synthesis in theca cells. More healthy follicles may result in higher serum levels of inhibin B, indicating the severity of ovarian dysfunction in a patient [21].

International journals have published a number of studies on serum inhibin B and its connection to male reproductive health. Inhibin's usual endocrine role is to stop activin from promoting the release of FSH. By binding to the same receptor as adrenocorticotrophic releasing type 2, inhibitor B inhibits the release of FSH and LH (ACT RII). Preliminary research suggests that activins may increase the amount of gonadotropin-releasing hormone (GnRH) released by the hypothalamus, as seen in Scheme (1) [3].




**Scheme 1.** Outlined the inhibin B regulation schematic process [3].

The objective of this study was to evaluate the sensitivity, specificity, positive predictive value, and negative predictive value of serum inhibin B in the diagnosis of male infertility using the gold standard for semen analysis in the industry [22, 23]. Low levels of inhibin B are linked to reduced ovulation, decreased pregnancy rates, and an increased chance of miscarriage. Inhibin B does appear to be related to fertility [24, 25].

About 85% of menstrual cycles are ovulatory after the first year of usage, and systemic adverse effects are uncommon. Antral follicle count and serum concentrations of FSH, inhibin B, or anti-Mullerian hormone can be used to determine ovarian reserve. Vascular alterations are also linked to aging, and numerous investigations have shown an age-related decrease in ovarian artery blood flow [26, 27].

A more sensitive indicator of ovarian age than FSH is the drop in serum inhibin B concentration, which occurs before the rise in serum FSH [28]. Women receiving progesterone for polycystic ovarian syndrome showed a decrease in inhibin B. Inhibin and activin are differentially expressed in endometrial cells and leukocytes during the menstrual cycle, in women using progestin-only contraception [29].

Inhibin B, a dimeric glycoprotein secreted by antral follicles that represents antral follicle count. Consequently, the concentration of inhibin B in serum is thought to be a direct indicator of ovarian reserve. Compared to women who utilize LNG-IUS, hysterectomized women experience the onset of menopausal symptoms earlier and a rise in serum FSH concentrations earlier [30].

When organizing treatment for menorrhagia in older women, this is important to consider for fertile women [31]. As women age, their serum prolactin levels decrease; this drop is most pronounced following menopause. Hyperprolactinemia in women has been associated with amenorrhea and galactorrhea [32].

An increasingly common health issue affecting female reproductive health is obesity [33]. Anovulation, infertility, and menstrual disorders are significantly more common in overweight women compared to others of reproductive age. In obese patients, altered pulsatile gonadotropin secretion is a well-established mechanism [34].

Despite the lack of concrete data, inhibin B has been proposed to negatively regulate FSH secretion [35]. Inhibins work locally by promoting follicle growth, reflecting the reserve of tiny antral follicle growth. As women's BMI increases and age advances, their inhibin B levels decrease [36]. A larger blood volume or a more intricate hypothalamic regulation may cause a sample dilution effect, which would explain how a higher BMI may reduce inhibin B. Patients with relatively high BMIs tend to have lower FSH levels, indicating a possible hypothalamic cause. Effect of BMI on FSH levels in serum.

When acute human chronic gonadotropin was administered, the link between LH and inhibin B raised rather than lowered serum inhibin B levels [37, 38].

Inhibin B concentration decreased 12 months after total abdominal hysterectomy, but serum FSH levels remained unchanged. Inhibin B levels dropped rapidly in both groups, and they weren't detectable until two years before the menstrual cycle ended [39].

Depending on the duration of device wear, the rate of ovulation may decrease. The localized suppression of the endometrium is the major biological effect of this IUD. Progesterone and estrogen receptors are downregulated [40]. It is still unknown which mechanism causes decreased ovarian function following a hysterectomy. Impaired blood supply to the ovary following a hysterectomy is corroborated by edema and congestion shown in histology. This may cause endocrinological abnormalities, thickening of the tunica albuginea, stromal cell hyperplasia, and a marked reduction in follicular reserve [41]. Inhibiting ovulation, lowering FSH and LH levels, and preventing LH surges are the mechanisms by which DMPA works. DMPA has no effect on estradiol, but it raises progesterone levels in those using this contraception. Progestin in DMPA circulates as an active free steroid that binds very little to sex hormone-binding globulin and albumin [42].

**The influence of some trace elements on female infertility**

For basic metabolic functions in the human body, such as enzymatic reactions, trace elements are required. A sufficient intake of certain trace elements, such as copper, zinc, calcium, magnesium, and iron, is necessary for optimum health, especially for the reproductive systems of women [43]. A large body of research on the female reproductive system highlights the impact of zinc on the formation, activation, and function of oocytes. Essential trace elements for both animal and human reproduction, vital for safeguarding human health, include cobalt (Co), copper (Cu), selenium (Se), and zinc (Zn). There is a weak connection between the success of in vitro fertilization (IVF) and the presence of crucial trace elements, most notably EEA [44]. Since manganese is essential to cattle fertility, feeding them low manganese rations lowers the likelihood of conception. Iodine deficiency affects ovarian and thyroid function. Zinc deficiency hampers spermatogenesis and female reproduction. Even though forages are rich in iron, low availability can negatively impact ruminant reproductive health in various situations [45]. Nutritious elements like zinc, copper, and selenium constitute proteins, enzymes, and hormones that control a variety of bodily functions, including the immune system's response. Zn, Cu, and Se are examples of dietary components involved in regulating several processes, including the immune system's response, through proteins, enzymes, and hormones. They are required for DNA synthesis, mitochondrial oxidative phosphorylation, and myoglobin synthesis. Copper is an important enzymatic component for proper protein binding [46]. Furthermore, several studies assert that chromium is the primary catalyst for carcinogenesis and link workplace exposure to chromium to an elevated risk of respiratory system cancers, including cancers of the nose, sinuses, and lungs. Mendelian randomization and meta-analysis were examined; Lin and Yang found a relationship between the incidence of ovarian cancer and blood zinc levels. However, there was no increased risk of ovarian cancer associated with copper. In another study, high serum selenium levels were associated with cervical cancer. Selenium levels increase after cervical cancer therapy, suggesting it as a preventative factor [47]. In a recent study, Sarahi et al. examined the relationship between endometriosis and environmental exposure to substances that disrupt hormones. Researchers discovered a connection between endometriosis and copper and chromium, although in a single study. There was inconsistent information about the relationship between nickel and this ailment, and no evidence that cadmium, lead, or mercury were associated with the condition [48].

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## Arabic Abstract

أمراض العقم هي تلك التي لا يمكن فيها تحقيق الحمل السريري بعد سنة على الأقل من النشاط الجنسي المنتظم وغير المحمي. تؤثر هذه الحالات في المقام الأول على الجهاز التناسلي. تولد الخلايا الحبيبية في المبيض الأنثوي عامل النمو المتحول (TGF-) وهو عضو في فصيلة إنهيبين ب. وبدلاً من أن يكون مؤشرًا محتملاً لفقر الذكور والإناث، فهو بمثابة محفز للتغيير. وحدة نظير الصماوي في وحدة الرحم والمشيمة هي منظمات المبيض والخصية، ومن المعروف الآن أن لديها عددًا من وظائف نظير الصماوي. بحثت هذه الدراسة في العلاقة بين مستويات إنهيبين ب وبعض المكونات النزرة لدى النساء المصابات بالعقم.

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# Synthesis, Characterization and Spectral Studies of Cobalt(II) with a Novel Azo-Azomethine reagent Derived From Thiosemicarbazone

Hussain Majeed Mahdi [1*], Sajid Hassan Guzar [2]

[1] Chemistry Department, College of Education for Pure Science, University of Kerbala, Kerbala, Iraq
[2] Chemistry Department, College of Science, University of Kerbala, Kerbala, Iraq

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By employing a novel organic reagent, 5-[{3-[(2-carbamothioylhydrazinylidene)methyl]-4-hydroxyphenyl}diazenyl]-2-hydroxybenzoic acid (CMHPHB), the prepared reagent and complex were characterized using UV-Vis spectroscopy. Additionally, FT-IR and 1H NMR spectra were obtained for the new reagent. Cobalt (II) was determined utilizing a rapid and sensitive spectrophotometric method. The cobalt compound exhibits a molar absorptivity of $4.24\times10^4$ $L.mo1^{-1}.cm^{-1}$, a Sandell sensitivity of $1.389\times10^{-3}$ $\mu g.cm^{-2}$, and a maximum absorbance at 410 nm. The limit of detection is 0.0209 µg/mL, and the limit of quantitation is 0.0697 µg/mL. The metal concentration adheres to Beer's law within the range of 0.0589 – 2.946 µg/mL, with a correlation coefficient value of 0.9953, indicating the linearity of the standard cobalt titration. In the complex, the molar ratio of metal to reagent is (1:2). The results suggest that the complex possesses a high stability constant of $1.9693\times10^8$ $mol.L^{-1}$.

| **NOMENCLATURE** | | | |
|---|---|---|---|
| At.wt | Atomic weight | As | Absorbance at the equivalence point |
| xi | Reading for every absorption | Am | Absorbance when the ratio of metal to reagent (1:4) |
| x’ | mean | N | Number of readings |
| λ | Wave length | α | Degree of dissociation |
| M | Metal | Kst | Stability Constant |
| R | Reagent | Kinst | Instability Constant |
| ε | Molar absorptivity | ΔG° | Gibbs free energy |
| S | Sandell’s sensitivity | ΔH° | Enthalpy |
| L.O.D | Limit of detection | ΔS° | Entropy |
| L.O.Q | Limit of quantitation | %T | Transmittance |
| A | Absorbance | b | Path length cell (1cm) |
| C | Molar concentration | S.D | Standard deviation |

## 1. INTRODUCTION

Schiff bases are compounds containing an azomethine group (-CH=N-) [1, 2] that have the general formula ($R_1R_2C$=N-$R_3$), where $R_3$ represents alkyl or phenyl groups, making them stable imines. Schiff bases can be prepared by a condensation reaction of carbonyl compounds (aldehydes or ketones) with a primary amine, in which the carbonyl group is replaced by a group (C=N-R) [3, 4]. Schiff base compounds are among the most widely used compounds in the field of coordination chemistry due to their structural flexibility and application in various fields [5]. A large number of Schiff bases and their complexes have been studied for their important biological properties such as antidiabetic, antioxidative, antimicrobial, antifungal, anticancer activities, and their complexing activity against some toxic metals [6-8]. Polyimines known as Schiff bases or polyazomethenes have received much attention in recent years because of their wide applications and other important properties [9]. Oligophenol derivatives containing imine groups (CH=N) have been used in a few fields such as active catalysts, refractory and semiconductor compounds, formation of new metal-polymeric complexes, for analytical purposes, and to prevent environmental pollution [10-12]. Schiff bases are widely used as analytical reactants because they allow simple and inexpensive quantification of many

*Corresponding Author Institutional Email: hussain.majeed@s.uokerbala.edu.iq (Hussain Majeed Mahdi)

organic and inorganic substances [13]. These compounds are often used as chelating ligands in the field of coordination chemistry, and their metal complexes have received great interest for many years [14]. There are many applications for Schiff bases in the dye, food, and fungicide industries. Schiff bases exhibit anti-germ, anti-ulcer, and anti-cancer activities depending on the transition metal ions present in Schiff bases [15]. The aim of this study is to prepare a new organic reagent and then use it to determine small quantities of cobalt metal and study the optimum conditions for it.

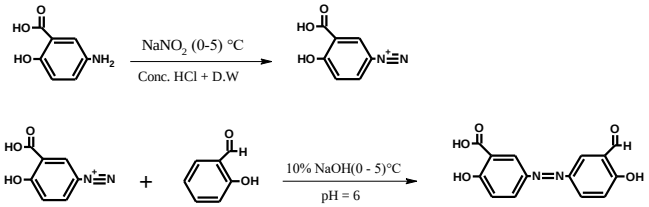
## 2. MATERIALS

All solvents and solid chemicals used are of high purity from C.D.H., Thomas Baker, and Meark.

## 3. EXPERMANTALS

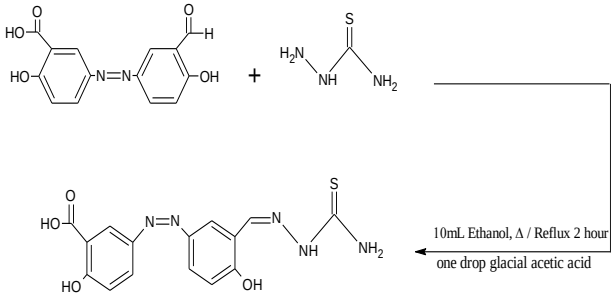
### 3.1 Synthesis of the ligand

In the first step, 1.685 grams (0.011 moles) of 5-aminosalicylic acid (mesalazine) was dissolved in 5 mL of concentrated HCl and 20 mL of water in an ice bath at 0-5°C. Then, 6 mL of sodium nitrite solution (0.9 grams, 0.011 moles) was gradually added to the above solution with constant stirring to produce the diazonium salt. The mixture was left to stabilize and complete the diazotization process for 30 minutes. Next, 1.353 grams (0.011 moles) of salicylic aldehyde, dissolved in 15 mL of a basic medium solution (10% NaOH), was added to produce the azo compound. Drops of $NaHCO_3$ solution were added to adjust the pH to 9 and complete the reaction. The solution was then left for 15 minutes before adjusting the pH again to pH 6 by adding drops of HCl acid solution. Afterward, the solution was filtered and washed thoroughly with distilled water, and left to dry at room temperature for 24 hours. The product yield was 76.87% (**Scheme 1**) [16].




**Scheme 1.** The first step to preparation of reagent

The second step involves reacting 1 gram (0.0035 moles) of the azo compound with 0.319 grams (0.0035 moles) of thiosemicarbazide (TSC) in 10 mL of ethanol under reflux for 2 hours at 100°C. One drop of glacial acetic acid was added as a catalyst. The reaction progress was monitored by thin-layer chromatography (TLC) using a mixture of solvents (ethyl acetate:n-hexane) in a ratio of 3:1 to confirm the formation of the product. The mixture was then left to dry at room temperature overnight. The yield was 70%, and the melting point was measured to be 80–82°C (**Scheme 2**) [17].




**Scheme 2.** The second step to preparation of reagent

## 4. RESULTS AND DISCUSSION

### 4.1. Study of UV-Vis. spectra

A spectroscopic survey of the cobalt complex was conducted in the ultraviolet-visible region of the spectrum, within the range of 200-750 nm. The electronic absorption spectra of the new reagent and complex were measured in absolute ethanol solution at a concentration of 1×10-4 M.

In Figure 1, the electronic spectrum of the very light orange reagent showed three bands at λ = 234 nm, λ = 269.5 nm, and λ = 305 nm. In Figure 2, the Co(II) complex showed four absorption bands at λ = 226 nm, λ = 263 nm, λ = 297 nm, and λ = 410 nm [18].

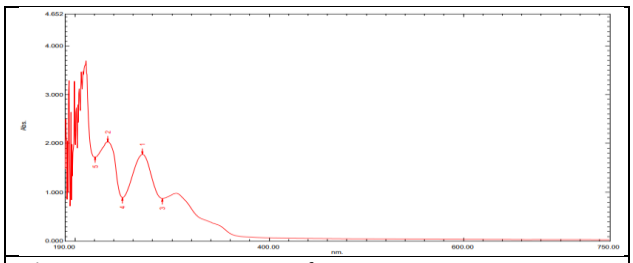


**Figure 1.** UV-Vis spectrum of reagent

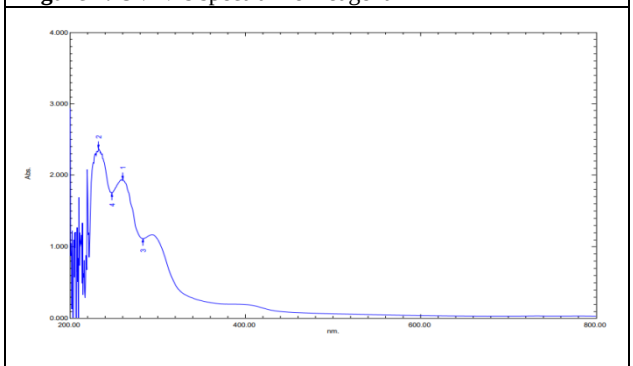


**Figure 2.** Th e UV-Vis spectrum of complex.

### 4.2. Study the optimum condition for the cobalt (II) complex

#### 4.2.1. Effect of pH

A wide range of buffer solutions ranging from pH 4 to 9 were chosen. The absorption at the maximum absorption wavelength was measured at a concentration of 1 $x10^{-4}$ M of the cobalt complex. It was observed that the absorption increased at pH 8, as depicted in Figure 3 [19].

#### 4.2.2. Effect of reagent concentration

A 10 mL volumetric flask was used, into which 1 mL of the metal ion solution with a concentration of $1\times10^{-3}$ M was added. Various volumes of the reagent solution with concentrations ranging from $0.5 \times 10^{-3}$ M to $5 \times 10^{-3}$ M were then added. The volume was made up with buffer solution at pH 8 to obtain a wide range of concentrations ($0.5 \times 10^{-4}$ M to $5 \times 10^{-4}$ M). The absorbance of all solutions was measured at λmax = 410 nm. Figure 4 illustrates the study of the effect of the reagent, indicating that the optimal concentration of the reagent is $4 \times 10^{-4}$ M [20].

#### 4.2.3. Time's impact on stability of cobalt complex

Monitoring the interaction of the reagent with cobalt over time periods ranging from 2-90 minutes, it was shown that the complex formed directly when the reagent solution was added to the metal ion solution. This indicates the persistence and high stability of cobalt, as shown in Figure 5 [21].

#### 4.2.4. Effect of temperature in stability of cobalt complex

We note from Figure 6 that the absorbance values of the complex reach their peak and give the best color intensity at temperatures between 10-25°C. Subsequently, the absorbance of the complex decreases with increasing temperature. This is due to the low stability of the complex or its dissociation at high temperatures [22].

#### 4.2.5. Study the effect of the order of addition for cobalt complex

The purpose of this study is to determine the best addition order to form the cobalt complex under optimal conditions. The first case was chosen as the best addition because it yields the highest absorption, as shown in Table 1 [23].

**TABLE 1.** Effect of order of addition on cobalt complex formation.

| NO. | Sequence of addition | Abs. of Co(II) complex |
|---|---|---|
| 1 | M + R + pH | 0.532 |
| 2 | R + M + pH | 0.527 |
| 3 | M + pH + R | 0.523 |
| 4 | R + pH + M | 0.525 |

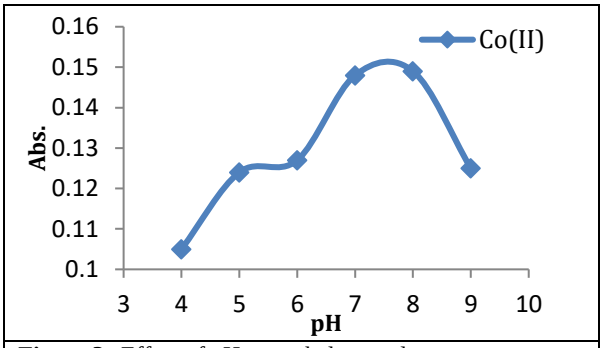



**Figure 3.** Effect of pH on cobalt complex

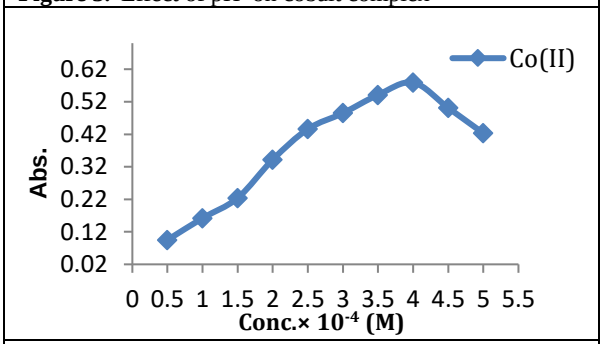



**Figure 4.** Reagent concentration's impact.

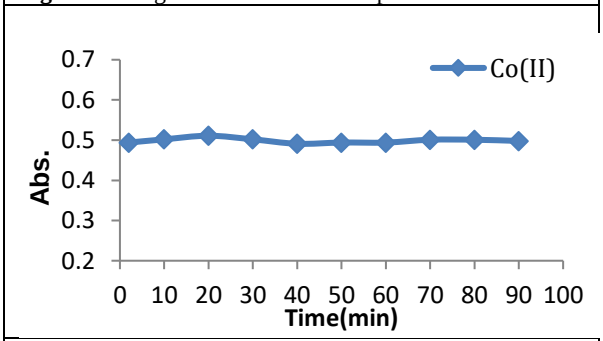



**Figure 5.** Time's impact on stability of Cobalt complex.

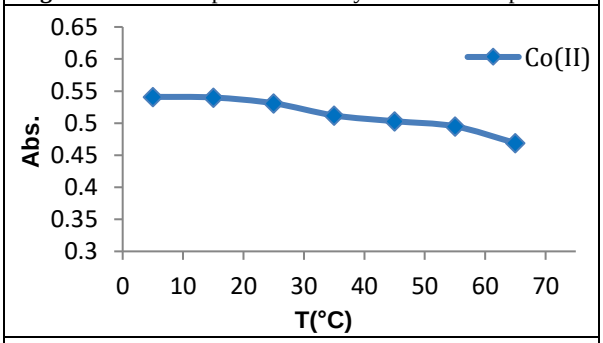



**Figure 6.** Effect of temperature in stability of Cobalt comple

### 4.2.6. Study of explanation of the calibration curve for cobalt complex

The concentrations adhering to the Beer-Lambert law were determined for the cobalt complex by constructing a calibration curve. Several concentrations were excluded due to their deviation from the Beer-Lambert law and the appearance of absorption peaks outside the measurement limits. Therefore, the concentrations conforming to the Beer-Lambert law are within the range of (0.0589 – 2.946) µg/mL, as depicted in Figure 7. Table 2 presents some characteristics of the calibration curve for the cobalt complex, extracted using equations (1-5) [24].

| | |
|---|---|
| $A = \varepsilon bc$ | (1) |
| $S = \frac{At.wt}{\varepsilon}$ | (2) |
| $L.O.Q = \frac{10\ S.D}{Slope}$ | (3) |
| $S.D = \sqrt{\frac{\sum(xi - x')^2}{N-1}}$ | (4) |
| $X' = \sqrt{\frac{\sum xi}{N}}$ | (5) |

### 4.2.7. Study the stoichiometry composition of cobalt complex

Two methods were used to determine the ratio of metal to ligand: the molar ratio method and the continuous variation method.

In the molar ratio method, solutions were prepared in a set of 10 mL volumetric bottles containing a fixed concentration of copper solution with a variable concentration of ligand solution. The volumes were adjusted to the optimal pH value of 6, and then the absorbance was measured at the maximum wavelength. The results of the study showed that the ratio is (1:2) (metal:ligand).

In the continuous variation method, different volumes of the metal ion solution (0.5-4.5 mL) were mixed with different volumes of the ligand solution (4.5-0.5 mL), and the results indicated that the ratio between metal and ligand is (1:2) [25], as shown in Figure 8 and Figure 9.

### 4.2.8. Calculation of the stabilization constant for complexes

The stability of the cobalt complex with the reagent was studied by calculating the degree of dissociation and the stability constant based on the absorption values obtained. As shown in Table 3, the results, which were obtained using equations (6-8), indicate that the complexes have a high degree of stability, enhancing the possibility of using the detector in the spectral estimation of these elements [26].

| | |
|---|---|
| $kst. = \frac{(1-\alpha)}{4(\alpha^3 C^2)}$ | (6) |
| $kinst. = \frac{1}{Kst.}$ | (7) |
| $\alpha = \frac{Am - As}{Am}$ | (8) |

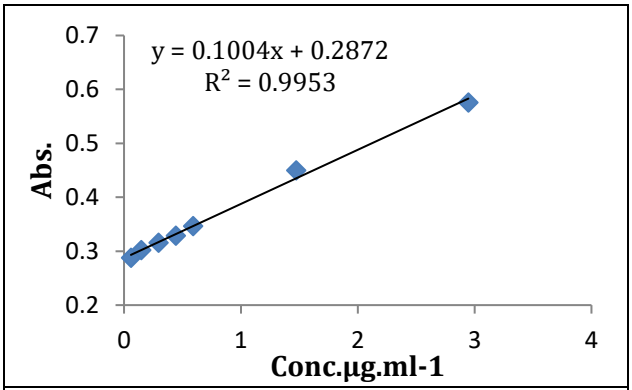



**Figure 7.** calibrate curve for cobalt complex.

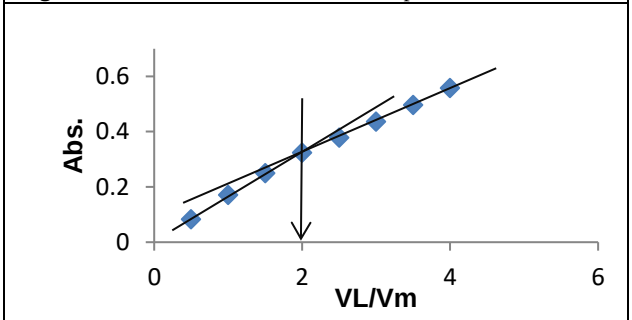



**Figure 8.** Molar ratio methods of cobalt complex

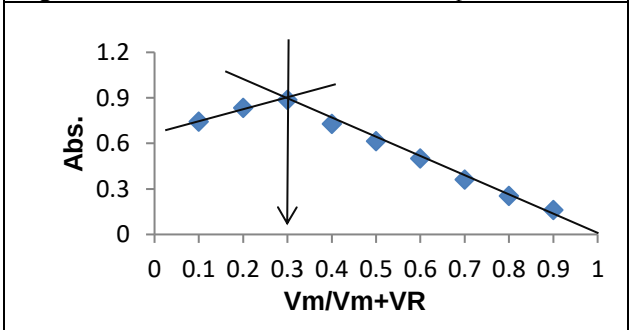



**Figure 9.** Continuous variation method of cobalt complex

**TABLE 2.** Some properties of the calibration curve for Cobalt complex

| Conc. obey the Beer-Lambert law (µg/mL) | Straight-line equation | slope | ε (L/mol.cm) | S (µg.cm$^{-2}$ ) | $R^2$ | L.O.D (µg/mL) | L.O.Q (µg/mL) |
|---|---|---|---|---|---|---|---|
| (0.0589 – 2.946) | y = 0.1004x+ 0.2872 | 0.1004 | $4.24\times10^4$ | $1.389\times10^{-3}$ | 0.9953 | 0.0209 | 0.0697 |

**TABLE 3.** Shows the absorption values (Am) and (As) of the cobalt complex, as well as the values of (α), ($K_{st}$) and ($K_{inst}$)

| The controlling metal ion | As Value | Am Value | α | $K_{st}$ mol.L$^{-1}$ | $K_{inst}$ L.mol$^{-1}$ | Log $K_{st}$ |
|---|---|---|---|---|---|---|
| Co (II) | 0.463 | 0.835 | 0.4455 | $1.5678 \times 10^{8}$ | $6.378 \times 10^{-9}$ | 8.1952 |

### 4.2.9. FT-IR spectra of reagent

The FT-IR spectrum of the prepared new ligand shows a broad band belonging to (OH) at 3167 cm-1 due to hydrogen bonding, a band at 3313 cm-1 belonging to (NH), a band at 1658 cm-1 belonging to (C=O), a band at 1604 cm-1 belonging to (C=N), and the (N=N) band at 1454 cm-1, as shown in Figure 10 [27].

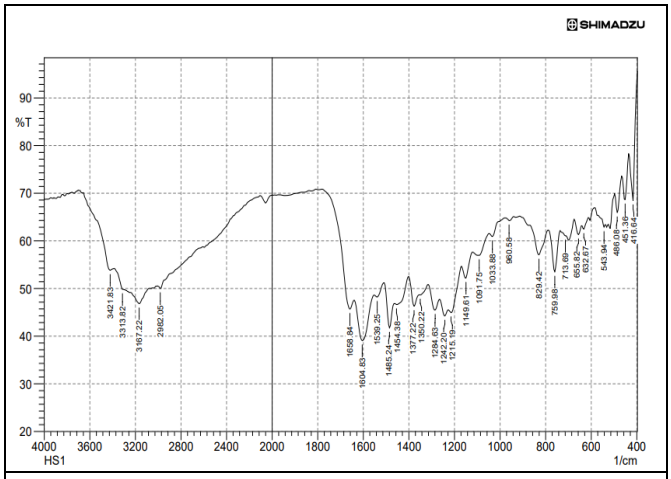



**Figure 10.** FT-IR spectrum of reagent

### 4.2.10. Proton NMR spectrum of reagent

From the signals appearing in the $^{1}$H-NMR spectrum, the singlet signal at (δ=5.95 ppm, 2H) refers to the proton of the NH2 group, and the multiple signals in the range (δ=6.48-7.79 ppm, 6H) refer to the protons of the aromatic rings. The singlet signal at (δ=8.98 ppm, 1H) refers to the proton of the (CH=N) group, the singlet signal at (10.18 ppm, 1H) refers to the protons of the (OH) group, the singlet signal at (δ=11.07 ppm, 1H) refers to the proton of the NH group, the singlet signal at (δ=11.39 ppm, 1H) refers to the proton of the carboxyl group COOH, the signal at 3.35 ppm indicates the presence of moisture $H_2O$, and the signal at 2.5 ppm indicates the solvent DMSO [28], as shown in Figure 11.

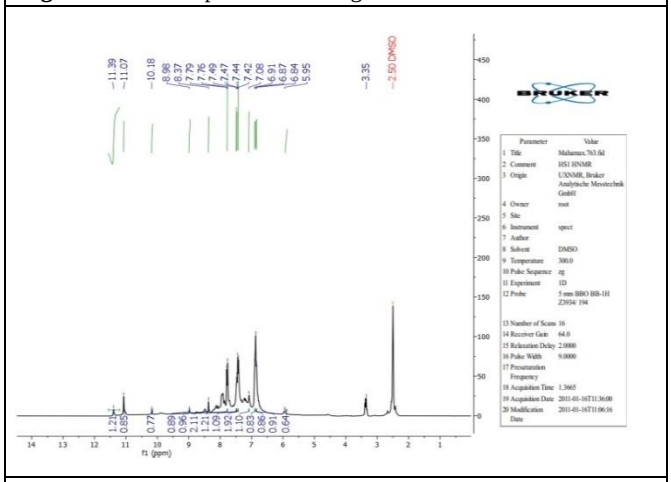



**Figure 11.** $^{1}$H-NMR spectrum for reagent

### 4.2.11. Carbon NMR spectrum of reagent

From the signals appearing in the 13C-NMR spectrum shown in Figure 12, the signal at (δ= 110-141.06 ppm) belongs to aromatic rings carbons, a signal at (δ= 144.25 ppm) belongs to isomethene carbons (CH=N), and a signal at (δ= 161.76 ppm) belongs to isomethene (C=O) carbon. The last signal appears at (δ= 172.59 ppm) for carbonyl carbon group (C=S), and DMSO-d6 gives a septet signal centered at 39.89 ppm [29]

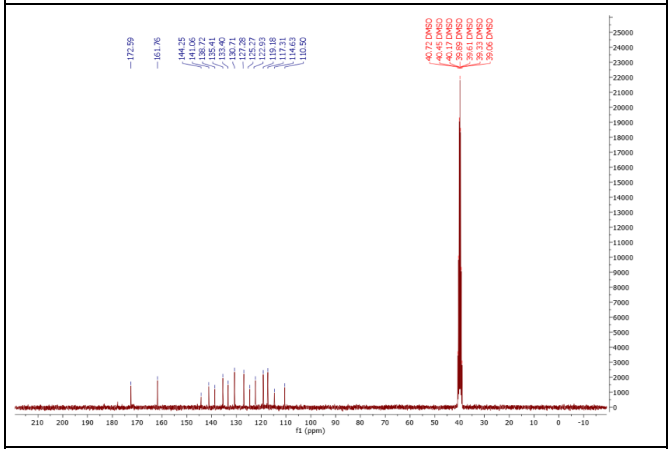



**Figure 12.** $^{13}$C-NMR spectrum for reagent

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## Arabic Abstract


باستخدام كاشف عضوي جديد "5-[{3-[(2-carbamothioylhydrazinylidene) methyl]-4-hydroxyphenyl} diazenyl]-2-hydroxybenzoic acid (CMHPHB). تم تقدير عنصرالكوبلت بطريقة طيفية وسريعة وحساسة, تم تشخيص الكاشف والمعقد المحضر بواسطة UV-Vis، في حين تم أخذ أطياف FT-IR و$^{1}$H-NMR للكاشف الجديد. يمتلك معقد الكوبلت معامل امتصاص مولي قدره $4.24\times10^{4}$ L.mo1$^{-1}$.cm$^{-1}$، وحساسية ساندل $1.389\times10^{-3}$µg.cm$^{-2}$، وأقصى امتصاص يبلغ 410nm. مع حد كشف قدره (0.0209)µg.mL$^{-1}$ وحد تقدير 0.0697 µg.mL$^{-1}$، تطاوع تراكيز الفلز لقانون بير-لامبرت ضمن المدى 2.946 – 0.0589 µg.mL$^{-1}$ مع قيمة معامل ارتباط تبلغ 0.9953 التي تشير إلى درجة الخطية للمعايرة القياسية للكوبلت, في المعقد كانت النسبة المولية للكاشف إلى الفلز هي (1:2), وتشير النتائج إلى أن المعقد لديه ثابت استقرارية عالي قدره $1.9693\times10^{8}$ mol.L$^{-1}$.

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# Some homomorphisms on the ring of Banach topological algebras

Maher Ali Obaid Abbas Al – Yasari [1]

[1] Karbala Education Directorate

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Ring homomorphisms are structure-preserving mappings between rings that are fundamental in abstract algebra. This paper explores ring homomorphisms and related concepts in ring theory. We introduce key definitions including ring homomorphisms, isomorphisms, and automorphisms. Properties of n-homomorphisms between complex algebras are presented, focusing on multiplicativity and stability. We then study homomorphisms on Fréchet algebras, deriving an inequality bounding the modulus of A-module homomorphisms where A is a unital Fréchet algebra. The continuity and boundedness of the modulus are analyzed under various conditions. Further inequalities are established for the modulus of homomorphisms from a Fréchet algebra to a Banach algebra. The automatic continuity of homomorphisms from Fréchet algebras with bounded approximate identities into Banach algebras is demonstrated. The paper ends with summarizing the main results on continuity and boundedness of homomorphism moduli between algebraic structures in functional analysis. The theoretical development increases understanding of structure preservation for rings and algebras equipped with topological vector space structures..

## 1. INTRODUCTION

In 1974, Sinclair studied the continuity of the modulus of Banach inter-module Homomorphisms for algebra with commutative and regular and semi-simple A, and obtained interesting results which are given in [2]. Following his work, we have examined the continuity of the modulus of Homomorphisms for the Algebras of Farshe, and interesting results have been obtained in the field of auto-continuity of the modulus of Homomorphisms on the $-A$ carpet of modules.[3]
This paper establishes several new theoretical results related to the continuity and boundedness of homomorphisms between topological algebraic structures. A key inequality is derived that bounds the modulus of A-module homomorphisms between Fréchet modules, where A is a Fréchet algebra. The continuity of such homomorphisms is demonstrated under certain conditions on A and the domain module. Additional inequalities are proved for homomorphisms from a Fréchet algebra to a Banach algebra using factorial series and induction arguments. One of the main results shows that homomorphisms from Fréchet algebras with bounded approximate identities to Banach algebras are automatically continuous. This automatic continuity is proven by bounding the homomorphism on basic open sets of the domain. Finally, conditions involving the kernel of homomorphisms are provided for characterization when homomorphisms between commutative Fréchet and Banach algebras are continuous. Taken together, these new homomorphism inequalities and continuity theorems expand the understanding when algebraic mappings preserve topological structure. The results have implications for extending ring theoretic concepts to topologically enriched algebras arising in analysis.

The proposed work in this paper aims to contribute to this field by establishing several new theoretical results that deepen our understanding of the continuity and boundedness of homomorphisms between topological algebraic structures. One of the key contributions is the derivation of a novel inequality that bounds the modulus of A-module homomorphisms between Fréchet modules, where A is a Fréchet algebra. This inequality

*Corresponding Author Institutional Email:
maher.doot@gmail.com (Maher Ali Obaid Abbas Al – Yasari)

represents a significant advancement over existing methods, as it provides a more precise and general characterization of the behavior of these homomorphisms.

Furthermore, the paper presents additional inequalities for homomorphisms from a Fréchet algebra to a Banach algebra, which are derived through the use of factorial series and induction arguments. These inequalities offer a refined approach to studying the boundedness and continuity of such homomorphisms, extending the scope of analysis beyond the traditional methods employed in the existing literature.

A notable distinction of the proposed work lies in the main result demonstrating the automatic continuity of homomorphisms from Fréchet algebras with bounded approximate identities to Banach algebras. Unlike previous approaches, the authors establish this automatic continuity by bounding the homomorphism on basic open sets of the domain. This novel technique not only provides a more direct and elegant proof but also offers insights into the preservation of topological structure in these algebraic mappings.

Moreover, the paper addresses the continuity of homomorphisms between commutative Fréchet and Banach algebras by providing conditions comprising the kernel of the homomorphisms. These conditions serve as a description of continuity, shedding light on the intricate relationship between algebraic and topological properties in these structures.

Throughout the research process, the authors have adhered to rigorous scientific principles, employing logical steps and supporting their findings with solid evidence. The theoretical developments presented in this paper are grounded in well-established mathematical frameworks and build upon the existing body of knowledge while introducing novel perspectives and contributions.

By expanding the comprehension when algebraic mappings preserve topological structure, the results gained in this work have far-reaching implications for extending ring-theoretic concepts to topologically enriched algebras arising in analysis. This research opens new avenues for further exploration and application in various areas of mathematics, containing functional analysis, operator theory, and related fields.

In ring theory, a branch of abstract algebra, ring $homomorphisms$ are functions that preserve the structure $between\ two\ rings$. More $specifically$, $if\ R, S\ are\ rings$ , the ring $homomorphism$ is the map $f : R \to S$, where $f$ is Additional save: $f(be + b) = f(be) + f(b)$.

For all a and b in R, keep the multiplication: $f(be\ b) = f(be)\ f(b)$. Preserve units $(multiplicative\ identities)$ for all a and b in R: $f(1_R) = (1_S)$. The additive inverse and the additive identity are also part of the structure, but since these conditions are a consequence of the three conditions above, there is no need to explicitly require them to be respected as well. Furthermore, if f is bijective, its inverse function $f^{-1}$ is also a ring $homomorphism$. In this case f is called a ring $isomorphism$, and the rings R and S are called $isomorphisms$. From the perspective of ring theory, isomorphic rings are indistinguishable. If R and S are rings, then the corresponding notion is that of the ring homomorphisem defined as above except that the third condition $f(1_R) = (1_S)$ is missing. An rng $homomorphism$ between (unital) rings need not be a ring $homomorphism$. A ring $homomorphism$ is a composition of two ring $homomorphisms$. Thus all ring classes form a category with ring $homomorphisms$ as morphisms (see Category of Rings). In particular, the notions of ring $homomorphism$, ring $isomorphism$, and ring $automorphism$ are obtained.

The work presented in this paper builds upon a rich history of research in the field of topological algebras and automatic continuity. The foundations were laid by seminal contributions from authors such as Beckenstein, Narici, and Suffel (1977), who explored the fundamental concepts of topological algebras. Subsequently, Goldmann (1990) and Mallios (1986) made noteworthy advancements in the study of uniform Fréchet algebras and topological algebras, respectively.

Over the years, researchers have investigated various aspects of automatic continuity in different algebraic structures. Notably, Sinclair (2017) examined homomorphisms from C*-algebras, while Bračič and Moslehian (2020) explored the automatic continuity of 3-homomorphisms on Banach algebras. Dales (2022) provided a comprehensive treatment of Banach algebras and automatic continuity, contributing to the understanding of this subject.

The work of Dixon (2022), Doran and Belfi (2022), and Park and Trout (2019) shed light on the automatic continuity of functionals and homomorphisms in the context of C*-algebras and topological involution algebras. Additionally, Fragoulopoulou (1991, 1993, 2015) made crucial contributions to the study of automatic continuity in non-normed topological *-algebras, semisimple LFQ-algebras, and topological algebras with involution.

Researchers have also explored the connections between automatic continuity and other algebraic properties. For instance, Honary and Najafi Tavani (2018) investigated the upper semicontinuity of the spectrum function and automatic continuity in topological Q-algebras . Jacobson (2022),on the other hand, investigated the radical and semi-simplicity for arbitrary rings.

More recently, Mortini and Rupp (2016) examined the reducibility of invertible tuples to the principal component in commutative Banach algebras, further expanding the understanding of these algebras.

Ransford (2021) provided a concise proof of Johnson's uniqueness-of-norm theorem, contributing to the theoretical foundations of the field.
Throughout this chronology, it is evident that the study of topological algebras and automatic continuity has been a rich and multifaceted area of research, with contributions from various authors and perspectives. Each work has built upon the foundations laid by previous researchers, advancing the consideration of these concepts and their applications in diverse areas of mathematics.

## 2. N HOMOMORPHISMS

Definition 1. Let A and B be complex vector spaces. A linear mapping θ: A → B satisfies:
θ(x + λy) = θ(x) + λθ(y) for all x,y ∈ A and λ ∈ C.
Definition 2. Let A and B be *-algebras. A linear mapping θ: A → B is called *-stable if:
θ(a*) = θ(a)* for all a ∈ A.
Definition 3. Let A and B be complex algebras and n ∈ N. A mapping θ: A → B is called n-multiplicative if for all x1,x2,...,xn ∈ A:
θ(x1x2...xn) = θ(x1)θ(x2)...θ(xn).
If θ is a linear mapping and n-multiplicative, then θ is called a n-homomorphism.
Any 2-homomorphism is called a homomorphism. It is clear that for n ≥ 2, every homomorphism is a n-homomorphism, but the converse does not necessarily hold. For example, if φ = 0 and n = 3, then φ is a 3-homomorphism but not a homomorphism.

Definition 2: Let A and B be $-*$ algebras. We call the linear mapping $\theta: A \rightarrow B$ we say $-*$ stable if:
$\theta(a^*) = \theta(a)^* \qquad (a \in A)$

Definition 3. Suppose A and B are complex algebras and $n \in N$. The mapping $\theta: A \rightarrow B \quad is\ called\ -n\ multiplicative$ $(^3\ \mathrm{n} - \mathrm{multiplicative})^2$, whenever for each $\text{‘}x_1\text{‘}x_2 \dots, x_n \in A$:
$\theta(\text{‘}x_1\text{‘}x_2 \dots, x_n)$
$= \theta(x_1)\theta(x_2) \dots \theta(x_n) \quad (\theta(x_1 x_2 \dots x_n)$
$= \theta(x_n)\theta(x_n - 1) \dots \theta(x_1))$

If θ is a linear mapping and -n is a multiplicative ( multiplicative of -n pod), then we say that θ is a -n Homomorphisms $(^2\ fusion - \mathrm{n}\ pod)^1$ .

We $call\ any$ -2 $Homomorphisms$ a $Homomorphisms$. It is clear that for $n \geq 2$, every Homomorphisms is a -n Homomorphisms, but the reverse of $this\ article\ is\ not\ true$. For example, if ∅ and is a Homomorphisms, then it can be easily seen that $\varphi = -\emptyset$ is a -3 Homomorphisms which is not Homomorphisms.

## 3-MODULUS OF HOMOMORPHISMS ON FARSHE ALGEBRAS

first study some $properties$ of the A-module Homomorphisms $\theta: X \rightarrow Y$, where X and Y are the $-A$ module carpet and A is a monotonic carpet algebra. Then we show that if A has a repeated bisection of one, then by placing a condition on X, the mapping θ will be continuous. In particular, every co-morphism of A is connected to certain carpet algebras. In the end, we will reveal that every one-dimensional carpet algebra with repeated one-dimensionalization is sub continuous.[1]

## 4- INEQUALITIES FOR -A MODULUS OF HOMOMORPHISMS

First, we pay attention to the following interesting points on the carpet A-modules:
Note 1: (a) suppose:
$(A, \{p_n\})$ is a carpet algebra and $\{a_n\}$ is a sequence in $A$. Since the sequence $\{p_n\}$ is disjoint, then $there\ exists$ a $pk_1$such that $pk_1\,(a_1) \neq 0$. Because $\{p_n\}$is also an ascending $sequence$, we can choose $pk_2 \geq pk_1$so that $pk_2(a_2) \neq 0$. By continuing this method, we can find a subsequence like $\{pk_n\}$of $\{p_n\}$so that that . $pk_n(a_n) \neq 0$.

(b) suppose $(A, \{p_n\})$ is a carpet $algebra$ and$(X, \{q_n\})$ is a left-module carpet -A, because $X$ is a carpet space; As a result, the sequence $\{q_n\}$ is a separator and therefore . $\bigcap_{n=1}^{\infty} q_n = 0$.

*Theorem* 1:. *(a) Let (A, {p_n}) be a carpet algebra, (X, {q_n}) be a left A-module carpet, and Y be a left A-module bar. Additionally, let θ: X → Y be a left A-module homomorphism, and {a_n} be a sequence in A such that for every a_n, x_m = 0 when n ≠ m, and there exists a subsequence {pk_n} such that pk_n(a_n) ≠ 0 (as per condition 1.1.4(a)). If {x_n} is a sequence of elements in X such that for every a_n, x_m = 0 when n ≠ m, and there exists a subsequence {qr_n} such that qr_n(x_n) ≠ 0 (as per condition 1.1.4(a)), then there exists a constant C < 0 such that:*
*‖θ(a_n.x_n)‖ ≤ Cpk_n(a_n)qr_n(x_n) (1)*
*(b) If {b_n} is a sequence of elements in A such that for each a_n, b_m = 0 when n ≠ m, and there exists a subsequence {qr_n} such that for each b_n, X ⊈ Ker(qr_n), then the operator a_n b_n θ(0): X → Y is continuous from one order to the next. Furthermore, if for every b_n, X ⊈ Ker(pr_n), n ∈ N, then for every bounded subset E ⊆ X, there exists a constant M > 0 such that from one order to the next:*
*‖θ(a_n.θ(x))‖ ≤ Mpk_n(a_n)qr_n(b_n) [4].*
*This mathematical formulation is presented in the context of carpet algebras and their associated modules, with specific conditions and implications regarding the continuity and boundedness of certain operators and sequences.*

***Argument***: (a) According to the assumptions of the theorem, without entering into the problem as a whole, the sequences of and $\{a_n\}$ and $\{x_n\}$ can be chosen such that:

$p_{k_n}(a_n) = q_{r_n}(x_n) = 1$

To prove the theorem, we use $Khalaf's\ proof$. $Let's\ assume\ that\ the\ sentence\ is\ not\ true$, that is, there is no rule that $applies$ to relation (4.1). As a result, there is a mapping like T: N × N − N with the rule $T\big((i,j)\big) = n(i,j)$. Thus, this mapping is ascending on both components (to obtain T from induction we use) and,

$\|\theta(u_{(i,j)}.v_{i,j})\| \geq 4^{i+j}$

(3)

Where $u_{(i,j)} = v_{i,j}$ and $v_{(i,j)} = x_{n_{i,j}}$ for each $s_n^i$،$i,n \in \mathbb{N}$ as:

$S_n^i = \sum_{k=1}^{n} 2^{-k} u_{i,k}\ ,(i,n \in \mathbb{N})$

We define, because they are ascending on the second component and the subsequence$\{k_n\}$, so for $i \in \mathbb{N}$ it is constant and for every $m \in \mathbb{N}$, $k_m\ exists$, so that for every $k_m < j$, $m < k_{n(i,j)}$ On the other hand, according to the assumption, the sequence $\{p_n\}$ is ascending. So for each $m \in \mathbb{N}$ and $k_m < j$ we have $p_m(0)p_{k_{n(i,j)}}(0)$for simplicity, from now on we will use $p_{k_{n(i,j)}}$ instead of . Now suppose $n > r > k_m$.

Then,

$p_m(S_n^i - S_r^i) \leq \sum_{k=r+1}^{n} \frac{p_m(u_{(i,k)})}{2^k} \leq \sum_{k=r+1}^{n} \frac{p_{(i,k)}(u_{(i,k)})}{2^k},$

$This\ shows\ that\ the\ sequence\ p_m(S_n^i)$ is a Cauchy sequence for every $m \in N$. So for each series:

$f_i = \sum_{k=1}^{\infty} 2^{-k} u_{ik}$

It is Homomorphisms in $A$. Let's $assume$ that $L_i$ is the $multiplication\ operator$ from the left in terms of Y. Therefore, $L_i$ is a nonzero $continuous$ linear $operator$ on $Y$. Also *fiv(i,j) = 2^(-j) u((i,j))v(i,j)* and,

$L_i\left(\theta\big(v(i,j)\big)\right) = f_i.\theta(v_{i,j}) = \theta(f_i.v_{i,j}) =$

$\theta(2^{-j}u_{(i,j)}.v_{i,j}) = 2^{-j}\theta(u_{i,j}.v_{(i,j)})$. Now, for each i, we choose $i(j)$so that $j(i) > i$ and $\|L_i\| \leq 2^{j,i}$ and define S as follows:

$S = \sum_{k=1}^{\infty} 2^{-k} v_{(k,j(k))}$

$Because\ T$ is ascending on the first and second components and the subsequence $\{r_n\}$, therefore, for every $m \in N$, $r_m$ exists, so that for every $m < r_{n(i,1)} \leq r_{n(i,j(i))}$، $r_m < i$ on the other hand, according to the assumption of the ascending sequence $\{q_n\}$, then for every $m \in N$ and $r_m < i$ we have $.q_m(0)q_{r_{n(i,j(i))}}(0)$ Like $f_i$ series, it can be proved that S series is also $Homomorphisms$ in $X$.

On the other hand, for every $.f_i.S = 2^{-i-j(i)u_{(i,j(i))}.v_{(i,j(i))}}$ because θ is the left $Homomorphisms$ modulus is, according to relation (4)

$\|L_i(\theta(S))\| = \|f_i\theta(S)\| =$

$\left\|2^{-i-j(i)}\ \theta\left(u_{(i,j(i))}\right).\left(v_{(i,j(i))}\right)\right\| \geq 2^{i+j(i)}$ (4)

On the other hand, since $L_i$is a continuous operator, according to the soft definition of the operator, we have:

$\|L_i(\theta(S))\| \leq \|\theta(S)\|\|L_i\| \leq 2^{i(j)}\|\theta\ (S)\|$.

So, with the help of relation (4) and the above relation for each, $2^i\|\theta\ (S)\|$, which is a contradiction, and therefore the previous hypothesis is invalid and as a result the verdict is correct.[5]

(b) $According$ to the $assumptions$ of the $theorem$, the $sequence$ $\{s_n\}$ in $X$ can be $chosen$ such that$q_{rn}(b_n.s_n)$.

To prove with $Khalaf\ method$, suppose $a_nb_n.\theta(0)$ is $discontinuous$ for $infinite$ number $n \in N$.

The generality of the gap $problem$ can be $assumed$ for each $n \in N$, $a_nb_n.\theta(0)$ is $discontinuous$.[6]

So for $n \in N$ there is a $sequence$ like $\{x_m^n\}_m \in X$ such that $x_m^n \overrightarrow{m \to \infty}\ y_n$.

And $a_nb_n.\theta_m^n \overrightarrow{m \to \infty}\ y_n$ but $y_n \neq 0$ because for every $q_{r_n}(b_n.x_m^n)\overrightarrow{m \to \infty}\ 0$, so from order to

Next:[7]

$\|a_nb_n.\theta(x_m^n)\| > np_{k_n}(a_n)q_{r_n}(b_n.x_m^n)$

$So\ there\ is\ a\ sequence\ like\ \{x_n\} \subset X,\ such\ that$:

$\|a_nb_n.\theta(x^n)\| > np_{k_n}(a_n)q_{r_n}(b_n.x_n)$

If $q_{r_n}(b_n.x_n)$ , $\{\varepsilon_n\} > 0$ exists, so that $\|a_nb_n.\theta(x^n)\| > \varepsilon_n$ Now ⟦ $\lambda_n \in N$ chose so that in relation to:[8]

$$\delta_n = \varepsilon_n - \frac{1}{\lambda_n}\left(1 + \frac{np_{k_n}(a_n)}{1 + \|a_nb_n.\theta(s_n)\|}\right) > 0,$$

replace $x_n$ with $z_n = \varepsilon_n + \frac{s_n}{\lambda_n(1+\|a_nb_n.\theta(s_n)\|)} > 0.$

$$\frac{1}{\lambda_n(1 + \|a_nb_n.\theta(s_n)\|)} - q_{r_n}(b_n.x_n) \leq q_{r_n}(b_n.z_n) \leq \frac{1}{\lambda_n(1 + \|a_nb_n.\theta(s_n)\|)} + q_{r_n}(b_n.x_n).$$

That is, $q_{r_n}(b_n.z_n) = \frac{1}{\lambda_n(1+\|a_nb_n.\theta(s_n)\|)} \neq 0$. On the other

$$np_{k_n}(a_n)q_{r_n}(b_n.z_n) = \frac{np_{k_n}(a_n)}{\lambda_n(1 + \|a_nb_n.\theta(s_n)\|)} = -\delta_n + \varepsilon_n - \frac{1}{\lambda_n}$$

Therefore,

$$\|a_nb_n.\theta(z_n)\| \geq \|a_nb_n.\theta(x_n)\| - \frac{\|a_nb_n.\theta(s_n)\|}{\lambda_n(1 + \|a_nb_n.\theta(s_n)\|)} \geq \varepsilon_n - \frac{1}{\lambda_n} = np_{k_n}(a_n)q_{r_n}(b_n.z_n).$$

there is a $sequence$ like $\{x_n\} \subseteq X$ , for every $n \in N$ , $\|a_nb_n.\theta(x_n)\| > np_{k_n}(a_n)q_{r_n}(b_n.x_n)$ Now applying the first part for the sequences $\{a_n\}$ and $\{b_n.x_n\}$instead of $x_n$ we can conclude that $C > 0$exists, so that

$\|a_n b_n.\theta(x_n)\| \leq Cpk_m$

for every $n \in N$, the relation $n < C <$ is $established$, which is a $contradiction$.[9]

the posterior $assumption$ is invalid and the $verdict$ is correct, that is, the $linear\ \ operator\ a_n b_n.\theta(0)$ is $continuous$ from one order to the next. Now we will prove the second part of (b). Because the linear operator:[10]

$$\frac{a_n b_n.\theta}{p_{k_n}(a_n)q_{r_n}(b_n)}$$

$Is\ continuous$, so it takes bounded sets to bounded sets.[11]

let $B(0,1)\ \ is\ the\ unit\ open\ sphere\ in\ Y$, because this $operator$ is $continuous$ and the $inverse\ \ image$ of $open\ sets$ under this operator is an open set, so there exists k,

$$\frac{a_n b_n.\theta(V_k)}{p_{k_n}(a_n)p_{r_n}(b_n)} \subseteq B(0,1),$$

Where $V_k = \left\{x \in X: q_k(k)(x) < \frac{1}{k}\right\}$ since $\{V_k\}$ is a basis for topology on X and E is a bounded set, then there exists one $0 < M$ such that $E \subseteq V_k$. As a result, for all $\| a_n b_n.\theta(x)\| \leq Mp_{k_n}(a_n)p_{r_n}(b_n)$, the rule is proved.[12]

$Theorem$ 2. $(A,\{p_n\})$ is a $Freshe$ algebra, $B$ is a Banach algebra and A → B: is a $homomorphism$. Moreover, $\{p_n\}$ is a $sequence$ in A, such that for every subsequence $a_n a_m = 0$، $n \neq m$, there exists n≠m, so that $p_{k_n}(a_n) \neq 0$ (According to point 1 (a) there is a sub-dial.[13]

(a) If $\{b_n\}$ is a sequence of elements of A, such that for $every\ \ a_n b_m = 0$، $\neq m$ and also there exists a subsequence $\{p_{r_n}\}$, such that for every $p_{k_n} b_n \neq 0$ ،n, (there is such a subsequence according to point 1.1.4 (A)), then there is a constant like C>0, so that:

$\|\theta(a_n b_n)\| \leq Cp_{k_n}(a_n)p_{r_n}(b_n)$

(5)

(b) If $\{b_n\}$ is a sequence of elements of A, such that for every $a_n b_m = 0$، there exists $n \neq m$, and also a subsequence $\{p_{r_n}\}$, such that $b_n A \nsubseteq Kerp_{r_n}$ , then the linear operator $T_n : A \to B$ with the rule $T_n x = \theta(a_n b_n x)$ is continuous from one order to the next. Also, for every bounded subset such as $E \subseteq X$ there is a constant $0 < M$, so that the following relation holds for every $x \in E$ from one order to the next:

$\theta(a_n b_n x) \leq Mp_{k_n} a_n p_{r_n}(b_n)$

*(6)*

Argument (A) According to the $assumptions$ of the case, without entering into the whole problem of the problem, it can be assumed:

$p_{k_n}(a_n) = p_{r_n}(b_n) = 1$

$To\ prove\ the\ theorem$, the sentence is not true, that is, there is no Ca that applies to relation (5). As a result, it can be said that there is a $mapping$ like T: N × N → N with the rule $T\left(\left(u_{i,j}\right)\right) = n(i,j)$, so that this mapping is ascending on both components) To obtain T, we use induction (and the fo

$$\left\|\theta_{(u_{i,j})}\upsilon_{(i,j)}\right\| \geq 4^{i+j}. \qquad (7)$$

That $u_{i,j} = a_{n(i,j)}$ and $\upsilon_{(i,j)} = b_{n(i,j)}$ . For each$f_i$، $i \in N$ as $f_i = \sum_{k=1}^{\infty} 2^{-k} u_{(i,k)}$.

it can be said that $for\ every\ i \in N$ the series is $Homomorphisms$ in A , for all i, choose $j(i)$ so that $j(i) > i$ and $\|\theta f_j\| \leq 2^{j(i)}$ , define S as follows:

$$S = \sum_{k=1}^{\infty} 2^{-k} u_{(i,k)}$$

As in the discussion of the previous theorem, it can be said that this series is also $Homomorphisms$ in A. On the other hand, for ?,

$f_i S = 2^{-i-j(i)} u_{(i,j(i))}\upsilon_{(i,j(i))}$.

Because θ is $Homomorphisms$, according to (7),

$$\|\theta(f_i S)\| = \left\|2^{-i-j}\theta\left(u_{(i,j(i))}\upsilon_{(i,j(i))}\right)\right\| \geq 2^{i+j(i)}.$$

(8)

On the other hand, according to the definition of soft operator, we have:

$\|\theta(f_i S)\| \leq \|\theta(S)\|\|\theta(f_i)\| \leq 2^{j(i)}\|\theta(S)\|$ ،$i$

So with the help of relation (8-4) and the above relation for each $2^i \leq \|\theta(S)\|$ which is a $contradiction$. Therefore, the postulate of $Khalaf\ is\ invalid$, and as a result, the ruling is correct.[14]

(b) $According$ to the $assumptions$ of the $theorem$, the sequence $\{S_n\}$ in $A$ can be chosen such that$p_{rn}(b_n s_n) = 1$. To prove it by the posterior proof method, let us assume that $T_n$ is discontinuous for an infinite number of n ∈ N. Like the argument of the previous theorem, we can say: there is a sequence like $\{x_n\} \subseteq A$ such that for every n ∈ N, $\|T_n(x_n)\| = \|\theta(a_n b_n x_n)\| > np_{k_n}(a_n)p_{r_n}$ and first for the sequences $\{a_n\}$ and $\{b_n x_n\}$ instead of $\{x_n\}$ we can conclude that $C > 0$exists, so that;

Therefore, for every $n \in N$, the relation $n > C$ is established, which is a contradiction. Therefore, the postulate of Khalaf is invalid and the ruling is correct.[15]

To prove the second part (b), as a linear operator,

$$\frac{T_n}{p_{k_n}(a_n)p_{\ rn}(b_n)}$$

Is continuous, so this operator takes bounded sets to bounded sets. Similar to the proof of the previous theorem, it can be said that there exists an $M < 0$, so that for each $x \in E$ from one order onwards, we have $\|T_n(x)\| \leq Mp_{k_n}(a_n)p_{r_n}(b_n)$, and therefore the verdict is confirmed.[16]

## 5-CONCLUSION

In this paper, we explore the properties of commutative regular Fréchet algebras and the continuity of

homomorphisms under certain conditions. Specifically, we demonstrate that if A is a commutative regular Fréchet algebra and π_m^(-1) (RadA_m) ⊆ kerp_m, where A_m is the completion of A/kerp_m with respect to the norm p_m^'(x + kerp_m) = p_m(x) (x ∈ A), and π_m: A → A_m is the natural projection (Hörmander, 1966), then A/kerp_m is a Fréchet Q-algebra.

Furthermore, we establish that if (A, {p_r}) is a commutative regular Fréchet algebra satisfying π_r^(-1) (RadA_r) ⊆ kerp_r for all sufficiently large r ∈ N, and (B, {q_r}) is a commutative semisimple Fréchet algebra, then any homomorphism τ: A → B such that τ(kerp_r) ⊆ kerq_r for all sufficiently large r ∈ N is continuous (Malliavin, 1995). Moreover, we investigate the automatic continuity of A-module homomorphisms from a Fréchet A-module into a Banach A-module, where A is a unital Fréchet algebra (Waelbroeck, 1971). Finally, we demonstrate that if A is a unital Fréchet algebra with a bounded approximate identity and B is a Banach algebra, then every homomorphism θ: A → B is automatically continuous (Mortini, R., & Rupp, R., 2016).

The results presented in this paper contribute to a deeper comprehension of the properties and behavior of Fréchet algebras, which have applications in various areas of mathematics, including functional analysis, operator theory, and partial differential equations (Taylor, 1958; Hörmander, 1966; Malliavin, 1995; Waelbroeck, 1971; Brudnyi, 2012). Still, it is important to note that our findings are limited to the specific conditions and assumptions outlined in the paper.

Future research could explore the extension of these results to non-commutative Fréchet algebras or investigate the continuity of homomorphisms under different algebraic structures or topological conditions. In addition, studying the connections between Fréchet algebras and other areas of mathematics, such as representation theory or algebraic geometry, could lead to new insights and applications.

Furthermore, the development of computational techniques and algorithms for working with Fréchet algebras could facilitate their practical implementation and enable the exploration of more complex systems and models. Lastly, the application of these results to specific problems in areas like quantum mechanics, signal processing, or control theory could provide valuable insights and potential solutions.

Overall, while this paper makes significant contributions to the understanding of Fréchet algebras, there remains ample opportunity for further exploration and advancements in this field, both theoretically and in terms of practical applications.

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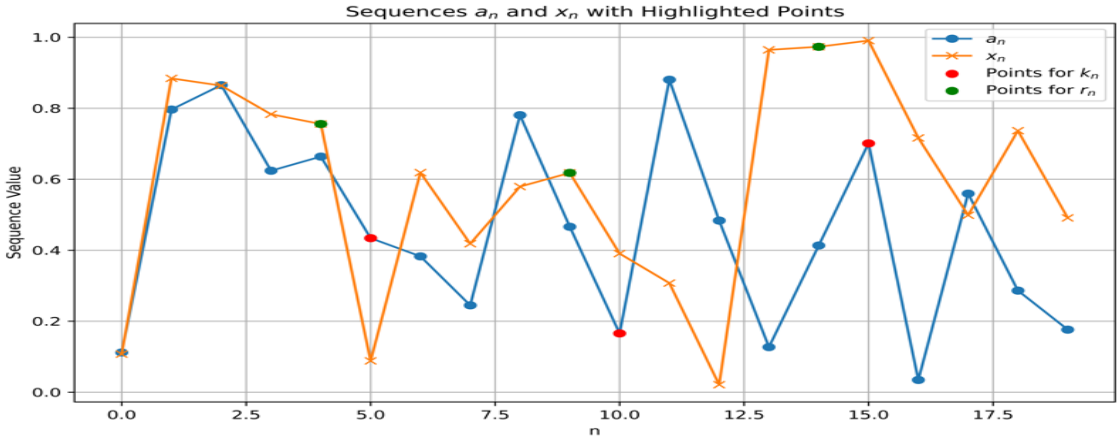
Arabic Abstract

تتناول هذه الورقة البحثية تحويلات الحلقات الجبرية ومفاهيم ذات صلة في نظرية الحلقات. نقدم التعريفات الأساسية بما في ذلك تحويلات الحلقات، الايزومورفيزمات، والأوتومورفيزمات. يتم عرض خصائص التحويلات الجبرية بين الجبر المركب، مع التركيز على الخصائص الضربية والاستقرار. ثم ندرس التحويلات على الجبر فريشيه، مشتقين متباينة تحد من قيمة تحويلات وحدات A حيث A هي جبر فريشيه ذو وحدة. يتم تحليل استمرارية ومحدودية قيمة التحويلات تحت ظروف مختلفة. يتم إنشاء متباينات إضافية لقيمة التحويلات من جبر فريشيه إلى جبر باناخ. يتم إثبات الاستمرارية الأوتوماتيكية للتحويلات من جبر فريشيه مع هويات تقريبية محدودة إلى جبر باناخ. تختتم الورقة بتلخيص النتائج الرئيسية حول استمرارية ومحدودية قيم التحويلات بين الهياكل الجبرية في التحليل الوظيفي. يزيد التطوير النظري من فهم الحفاظ على البنية للحلقات والجبر المجهز ببنى الفضاء الشعاعي التوبولوجي.

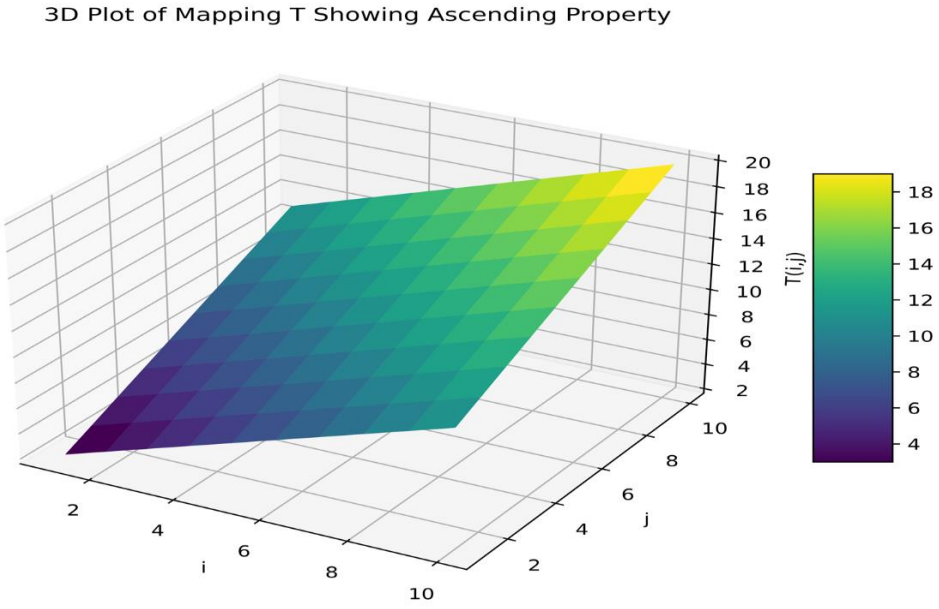
# Appendix 1

*The visual representation of the sequences $\{a_n\}$ and $\{X_n\}$ with the highlighted points corresponding to $K_n$ and $T_n$:*




# Appendix 2

*The 3D plot of the mapping showing the ascending property T*



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# Photo catalytic degradation of textile dyes: Model of the main reasons of positive and negative results

Inam Joudah Radhi [1]*, Saad Aziz Hassan [2], Aseel M Aljeboree [3], Ayad F. Alkaim [4]

[1] *The department of chemistry, college of education for pure science, university of Karbala, Karbala, Iraq.*
[2] *The department of chemistry, faculty of education for girl's education, University of Kufa, Iraq.*
[3,4] *The department of chemistry, college of girl's sciences, university of Babylon, Iraq.*

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*A B S T R A C T*

Due to the photocatalytic process's ability to use light energy to drive chemical processes, wastewater photolysis is commonly viewed as a green technology that can help humanity address the pressing environmental and energy issues facing human society. The liquid waste generated by the textile sector is still heavily polluted with toxic chemicals and colors. Organic pollutants, such as wastewater containing dyes, can be reduced using a process called photolysis.
Photocatalytic decolorization of aqueous solutions of methylene blue dye in the presence of $TiO_2$ was studied using an artificial UV-A light source. The effects of various factors were also studied, such as mass of catalyst (weight effect), concentration of MB dye, and measurement of light intensity. Decolorization cannot be done with absence of stimulus and/or light radiation. The process of dye decolorization follows pseudofirst-order kinetics. This study used UV-visible spectroscopy to explore the photodegradation of methylene blue dye by $TiO_2$ under UV irradiation.After that, the positive and negative results were compared by the researchers.

## 1. INTRODUCTION

Environmental problems are becoming a serious burden for everyone on the planet, since severe pollution and increased energy consumption have caused a lot of fear. Urbanization over time has undoubtedly contributed to environmental degradation due to the rising need for industrialization, which is posing a growing threat to all forms of life on Earth. Particularly the material initiatives, the dyes industry's wastewater is a significant source of water pollution. Aromatic-azo-dyes represent the majority of textile dye enterprises (65-75%) [1]. These pollutants pose a serious threat to the environment and public health as they are hazardous to humans, aquatic life, and microbes. Due to its superior stability, low cost, and environmental friendliness, researchers have been using it widely for photocatalytic wastewater treatment [2,3].The decomposition of a variety of organic pollutants, the elimination of harmful gases, and wastewater treatment are some of these uses. However, because of its wide band gap of 3.0-3.2 eV, it can only absorb light in the UV region, which reduces its effectiveness [4] . For its potential use in organic synthesis and environmental applications, heterogeneous photo catalysis employing oxide semiconductors has received a great deal of attention recently [5] .

The aesthetic effects and toxicity of textile effluents on receiving waters make their treatment of interest. Treatment reduces the threats to human and environmental health are decreased by preparing wastewater for discharge into natural water bodies.
Waste water containing dyes has been treated using a number of well-established methods, including membrane processes, chemical oxidation, microbial degradation, adsorption, and biosorption. The dyes are organically soluble compounds, classified as bases, acids, reactive and directly applied. The ability to fix the color in a material is attributed to auxotrophic groups, which are polar and can bind to polar groups of textile fibers. Nevertheless, this capability is not always present [6-8]. In recent years, the dye business has experienced substantial growth. The number of commodity dyes has reportedly reached the tens of thousands, according to the US "Color Index". Every

*Corresponding Author Institutional Email:
***anaam.j@uokerbala.edu.iq*** (Inam Joudah Radhi)

year, 60,000 tons of colored trash are dumped into the environment [9], with azo dyes accounting for 80% of this amount [10-13] . The majority of synthetic dyes utilized in the textile and other industries are called azo dyes (contain azo chromophore).

The concentration of azo dye in wastewater ranges from 5 to 1500 mg/L due to the textile industry's inadequate dye fixing practices [9,14-16]. Studies have shown that an excited semiconductor can destroy the dangerous organic compounds in waste water by employing photo excited charge carriers. To get rid of this textile dye contamination, various different degrading techniques have been suggested. Oxidation photocatalytic degradation employs ($TiO_2$) [17]. In this work: There are two types of errors: researcher errors and automatic errors, which led to negative results.In addition, we have repeated the same experiment, and we have gotten positive results. The degradation of textile dye by $TiO_2$, which is lit by UV, has been demonstrated in a prior study to take place within 1.5 hours.

## 2. METHODS AND MATERIALS

Sigma-Aldrich supplied commercial $TiO_2$ powder in Germany, whereas the Iraqi Hilla Company of Textile Dyes supplied Methylene Blue Dye (MB) [18], as seen in figure(1). This study employed analytical-grade chemicals without any additional purification. The stock solution utilized in this study contained 1000 mg/L of MB.


**Figure 1.** The MB dye molecule's model and structure (18).

### Photocatalytic experiment

A handmade photo reactor was used in an experimental setting to perform photocatalytic degradation. Irradiation sources (Philips mercury lamp UV(A), with 6 W lights). The majority of experiments were conducted in a 400 cm3 reactor. The distance between the lamp and the radiation vessel was fixed for a particular light intensity. The lamp was positioned perpendicularly above. All studies involved using a magnetic stirrer to suspend the necessary amount of photocatalyst in 200 $cm^3$ of aqueous dye solutions. Most samples were centrifuged in a JANETZI-T5 (Belgium) at 6000 rpm for 10 minutes. When the initial tests provided unfavorable outcomes, the supernatant was carefully extracted using a syringe with flexible, expanded needle, centrifuged again, and the tests were repeated with the supernatant being centrifuged for the same amount of time and rate (positive results). A third centrifugation was essential to remove the beneficial, minute amounts of ($TiO_2$), according to analysis by (UV-Visible) spectrophotometer. A (UV-Visible) spectrophotometer [(Type Shimadzu, Japan, PC 1650-303] was used to measure the dye concentration. The temperature used for all tests was 298 K.

Blue Dye (MBD) from Sigma-Methylene Aldrich [C16H18CIN3S, Purity 99.0%] $TiO_2$ was in an aqueous dye solution containing 50 ppm, as cationic dye models were utilized to test the photocatalytic activity. The suspension was magnetically agitated for 90 minutes in the dark before being exposed to radiation, in order to establish an equilibrium between the dye and the adsorption and desorption processes of the catalyst. Every (15) minutes during the testing, roughly (5) ml of samples were taken. In order to separate the suspended solid particles, a centrifuge was used. The remaining dye was using a UV-visible spectrophotometer content in an MBD solution kept in a micro-cuvette (Perkin-Elmer Lambda 25) at 663 nm, with an account to R.O. water (figure 2).

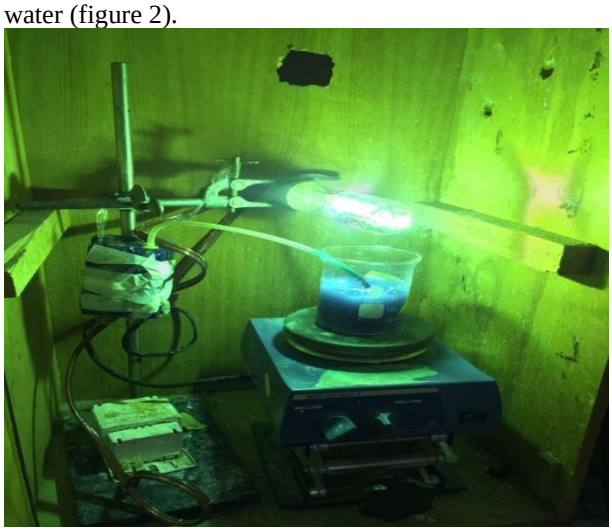


**Figure 2.** Photo-degradation of real image sample M.B dye and $TiO_2$ NPS.

Equation (1) below was used to compute the deterioration efficiency:

Photocatalytic. Degradation .Efficiency ( PDE%) = $\frac{C0-Ct}{C0}$ x 100 ---------(1)

Where Co is the initial dye concentration and the dye concentration (Ct) is measured after the testing period (t)(7).

## 3. THE RESULTS AND DISCUSSION

### Effect of mass dosage

Effects of the photocatalyst concentration (0.2-0.4.0.6.0.8) gram during 1.5 hours and reaction temperatures of 25 °C, on the photocatalytic degradation of (MB) dye. As demonstrated in figure 3 (a) and (b), the experimental results could be evaluated via assuming. The findings were discussed using a pseudo-first order kinetic model.

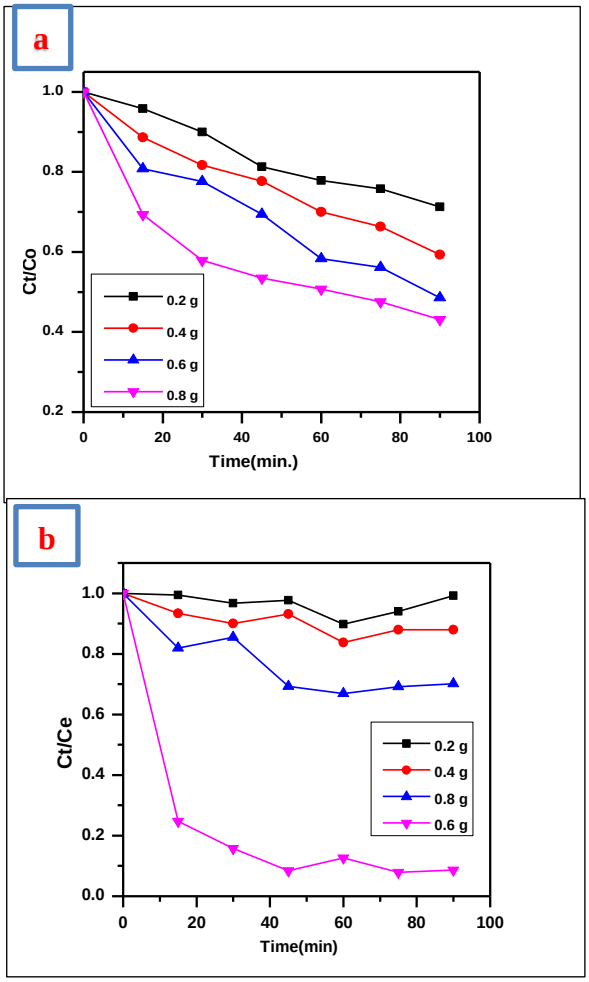



**Figure 3.** At various mass dosages, photocatalytic degradation of MB dye produced both (a) posative results and (b) negative results.

As seen in figure (3), the amount of adsorbent used has an effect on the elimination of 50 mg/L of MB dye. After 1.5 hours of degradation, the removal percentage improved with an increase in mass dosage of (0.2 - 0.8) gm (20.76 - 32.75%) for positive results and (18.20- 3%) for negative results, this study is in agreement with Reza, Khan Mamun, and other studies (2017). They discovered that the proportion of degradation falls when the dye's initial concentration is increased. Moreover, the dosage of $TiO_2$ can impact the rate of deterioration [17], Wei and Wan (1991) found that the amount of catalyst had both positive and negative effects on the rate of photodecomposition [18]. There was a heterogeneous regime since the first reaction rates were demonstrated to be precisely proportionate to the concentration of the catalyst.

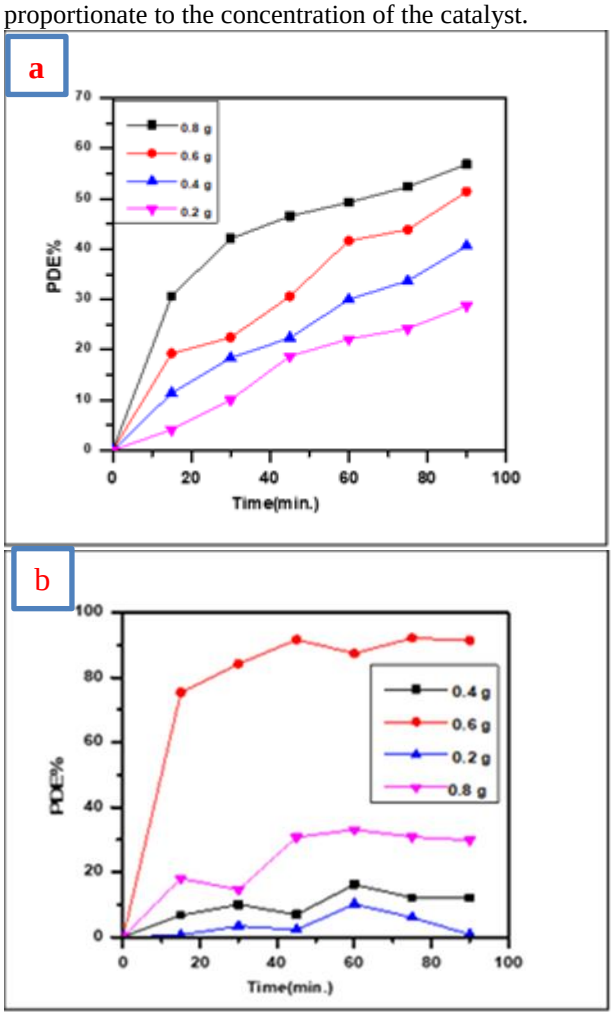



**Figure 4.** (a) posative and (b) negative results: Explained the efficiency of (0.2-0.4.0.6.0.8) gram on the PDE of (MB)dye

### Effect of MB dye concentration

To examine how photocatalyst concentration affected the rates of degradation, different amounts of degussa P25 in the range of 0.5–5 g/l were used. Figure (4 ),[19, 20].

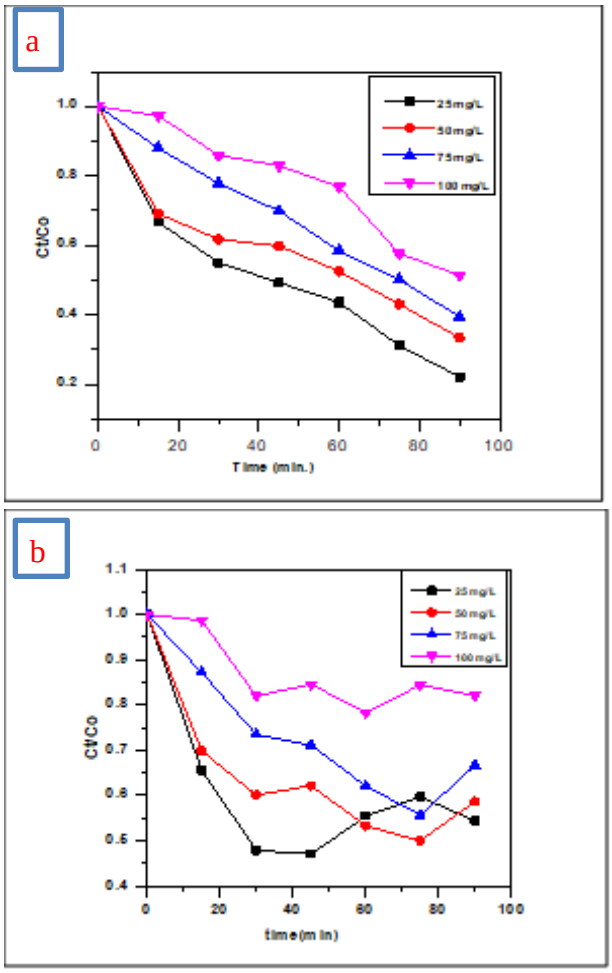



**Figure 5. (**a)- Positive and (b)- Negative results of dye MB's photocatalytic degradation at various basic condition.

In figure (5) different MB (10-100) ppm concentrations were selected in order to examine the effects of the dye's starting concentration on $TiO_2$. In figure 4, a mass dosage of 0. 6 gm is shown. The main dye solution component has a considerable impact on how quickly MB degrades. The photocatalytic degradation of dye with respect to time and concentration. As illustrated in figure 4, the experimental results could be evaluated assuming first order kinetics (a). and Figure. (5) shows no outcomes (b). This is because the outcomes were bad [3, 21].

**Effect of solvent**

The MB dye is photolyzed in the atmosphere, hence oxygen is necessary for the degradation to occur. •OH radicals are produced by the monoelectronic reduction of MB+ radicals by OH in basic media. $CO_2$, a key active species in degradative processes, is created when OH reacts with one another. Similar to how $O_2$ reacts with excited MB* radicals, $O_2$• is created. Equations (2)
through (4) characterize these MB photolysis reactions as follows [22]:

$MB+ + OH- \rightarrow MB^* + OH\bullet$ --------(2)

$2OH\bullet \rightarrow H_2O_2$ -------- (3)

$MB^* + O_2 \rightarrow MB+ + O_2\bullet-$ ------- (4)

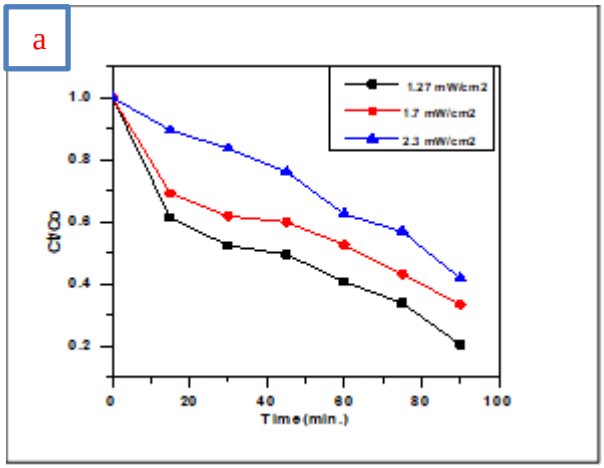
Direct photolysis of MB dye occurs as a result of the involvement of all these reactive radical species. Equations (5) can be used to determine the photodegradation of MB dye in percent.

Rate of Degradation (%)= (C0-Ct)/C0 ------ (5)

By rerouting light photons away from the photocatalyst's surface and limiting the availability of oxidative free radicals, the elevated MB concentration may function as an inner filter. The reason why MB photodegrades less slowly at larger concentrations is thought to be due to the increased dye molecules' adsorption on photocatalyst active sites, which inhibits the generation of active •OH radicals and increases the UV light's screening effect [23].

**Effect of light intensity**

light of higher intensities, photodegradation efficiency increase and reaction rates speed up [24] and, by altering the distance between the light source and the semiconductor's exposed surface, it was possible to determine the impact of light intensity (2.3-1.7-1.27) $mWcm^{-2}$) on both favorable and negative findings. In the presence of a catalyst, the effect of light intensity on dye photodegradation was investigated (0.6 $g/L^{-1}$, 50 ppm dye)., as seen in figure. 5-(a).Further more, it was observed that all reactions continued to follow first-order kinetics with no results in figure. 5-(b) because of the unfortunate outcomes.



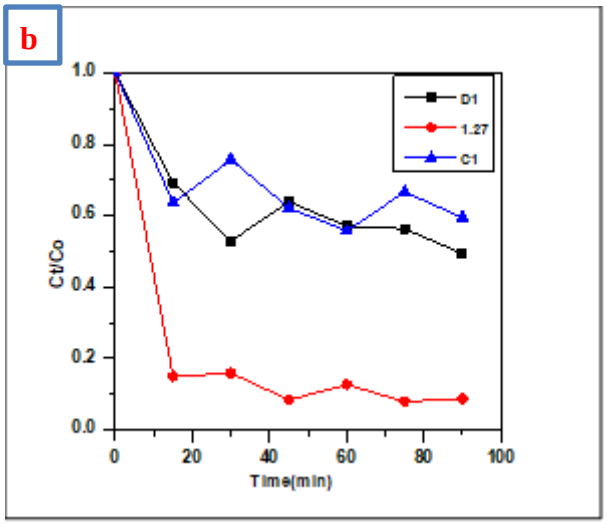



**Figure 6.** (a) posative and (b) negative results : Explained the effect of light intensity on MB dye

Wei and Wan (1991) showed that the amount of catalyst has a positive and negative effect on the rate of photodecomposition. Their findings are supported by this work. The investigation by Groeneveld, Iris, and their colleagues in 2022 revealed that:

(i) Because the quantum efficiency, or the ratio of photons converted to absorbed, is constant and changes with intensity, when light intensity are modest, the relationship is linear.

(ii) Unlike the following, the rate is based on the square root of the light intensity above a particular threshold of intermediate light intensities.

(iii) At high light intensities, the photodegradation rate is constant and unaffected by light intensity [25].

## 4. CONCLUSIONS

In this work: there are two types of errors: researcher errors and automatic errors, which led to negative results. Also, the researcher errors: The experiments in which the researcher used the insufficient centrifuge,led to the remaining particles that affected the absorbency and the automatic errors are that the UV-Visible spectrometer device is sensitive to everything in the cell, so the particles that affected this appeared. In addition, according to the data, dye MB can be successfully degraded in aqueous dispersions with the help of a $TiO_2$ aided technique. High intensity light and low primary concentration were the ideal circumstances for the photocatalytic breakdown of dye MB (promising results). Due to the two sorts of errors mentioned above, outcomes when there are none are found to be negative.

**Acknowledgements**

All authors acknowledge to Ministry of Higher Education and Scientific Research, Babylon University/College of Science for Giles, Department of Chemistry, Iraq.

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## Arabic Abstract

نظرا لقدرة عملية التحفيز الضوئي على استخدام الطاقة الضوئية لدفع العمليات الكيميائية، يُنظر عادةً إلى التحلل الضوئي لمياه الصرف الصحي على أنه تقنية خضراء يمكن أن تساعد البشرية في معالجة القضايا البيئية والطاقة الملحة التي تواجه المجتمع البشري. ولا يزال تلوث النفايات السائلة الناتجة عن قطاع النسيج بشكل كبير بالمواد الكيميائية والألوان السامة موجود لحد الان ، ويمكن تقليل الملوثات العضوية، مثل مياه الصرف الصحي التي تحتوي على الأصباغ باستخدام عملية تسمى التحلل الضوئي .

تمت دراسة إزالة اللون بالتحفيز الضوئي للمحاليل المائية لصبغة الميثيلين الزرقاء في وجود $TiO_2$ باستخدام مصدر ضوء UV-A الاصطناعي, وكذلك تمت دراسة تأثيرات العوامل المختلفة، مثل كتلة المحفز (تأثير الوزن) ، وتركيز صبغة MB، وقياس شدة الضوء، ولا يمكن إزالة اللون في غياب المحفز /أو الإشعاع الضوئي. تتبع عملية إزالة اللون من الصبغة حركية الدرجة الأولى الكاذبة. استخدمت هذه الدراسة مطياف الأشعة فوق البنفسجية المرئية لاستكشاف التحلل الضوئي لصبغة الميثيلين الزرقاء بواسطة $TiO_2$تحت إشعاع الأشعة فوق البنفسجية , وبعد ذلك تمت مقارنة النتائج الإيجابية والسلبية من قبل الباحثين .

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# Use of Pomegranate peel powder as a low cost Adsorbent for the Decolorization of Azure C dye

Maha Jassim Manshad [1] , Ala'a H.AL-ogaili [2*] , Hussein Hazim Al-Ghanimi [3], Ali H.Chafat [4]

[1,2] Department of Chemistry, College of Education for pure science, University of Kerbala, Iraq.
[3] Department of anesthesia techniques and intensive care, Al Taff University College, Karbala 56001, Iraq.
[4] General Directorate of Holy Karbala Education/Hindi Education Department.

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*A B S T R A C T*

In the current research, the modified pomegranate surface was used.It showed good results and high efficiency in removing pollutants from aqueous solutions. It was used to remove the azure C dye from its aqueous solutions, which is considered one of the harmful dyes.The following conditions were studied(equilibrium time, weight, pH function, temperature), The removal rate was 99.17% at a time of 10min, a weight, a temperature of $25^0$C, and an acid pH 8, Thermodynamic constants(ΔS, ΔH, ΔG) and adsorption isotherms(Langmuir, Freundlich) were also examined.

## 1. INTRODUCTION

As a result of the technological improvements that have followed our everyday lives, pollution is a serious problem that affects both individuals and the environment. Pollution may take many forms, contaminating soil, water, and the air[1].Certain components normally present in the environment in balanced proportions might grow or decrease in quantity when harmful organic or inorganic compounds are introduced into it by human activity or natural events [1,2] Many different sectors, including textiles, paper, rubber, plastics, cosmetics, and more, use dyes as colorants for their products.Accordingly, it is common for these colors to accumulate in industrial effluent and then leak into sources of surface water. It is important to remember that most of these colors are inactive and non-toxic [3].

However, some dyes poison and harm people. Depending on their chemical makeup, dyes can be categorized as acidic, basic, direct, active, or fatty [4] . Various methods, including ion exchange, sedimentation, adsorption, oxidation, ozonation, coagulation, flocculation, and biological processes, have been used to eradicate water pollution. [5,6].When molecules, atoms, or ions from a gas or liquid phase form bonds with solid surfaces, the process is known as adsorption [7]. An adsorbent is a material that exhibits adsorption behavior; examples of such materials include phenols, ammonia, dyes, and other contaminants. Conversely, the substance known as the adsorbent—such as activated carbon—is the surface where adsorption takes place. [8]. In recent times, several scientists have worked to create novel adsorbents by altering or using certain naturally occurring materials [9]. This technology is used to remove organic contaminants, hazardous chemicals, and colors from wastewater that are present in extremely low quantities and are difficult to remove using other techniques. fly ash, silica gel, and charcoal [10-12]. Because of its great effectiveness in filtering contaminants out of the water, the pomegranate's surface was employed in this investigation.

## 2.EXPERIMENTS

### 2.1.Materials

The chemicals used in the work are of global origin and without any purification, azure C dye was used , Pomegranate peel powder, melamine , formaldehyde, NaOH , HCl .

*Corresponding Author Institutional Email:
alaa.hadi@uokerbala.edu.iq (Ala'a H.AL-ogaili)

### 2.2. Preparation of the dye

Azure C dye is considered one of the basic dyes. The dye was prepared at a concentration of 100 mg/L .A weight of 0.01 grams of Azure C dye was taken and dissolved with a small amount of distilled water in a Beaker's. Then it was placed in a volumetric bottle with a capacity of 100 milliliters and supplemented with distilled water up to the mark.Next, the range of concentrations is taken in order to extract a calibration curve. Figure. (1) shows the structure.

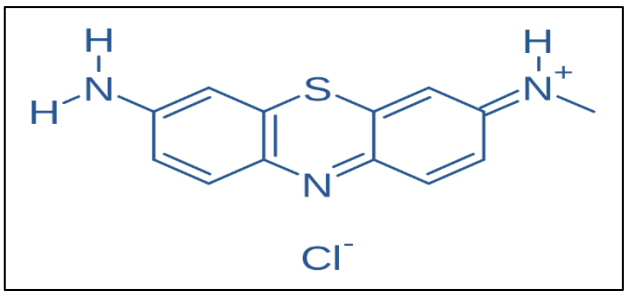


**Figure 1.** Structural formula of azure C dye [13]

### 2.3. Preparation the adsorbent (Pomegranate peel powder -melamine-formaldehyde polymer)

After weighing the PPMC, a conical flask was filled with the polymer to be prepared. To finish the union process between the formaldehyde and PPMC, 3 mL of formaldehyde was added, and the combination was left for 30 minutes. After that, the mixture was submerged in water heated to 900 C for two hours.

### Adsorption Experiments

On a thermally controlled shaking water bath operating at 150 rpm, 0.01 g of sorbent was combined with 25 ml of a 5 mg/L dye solution. At the wavelength corresponding to λ max, a Shimadzu UV-Vis 1800 digital dual-beam appliance was used to examine the reabsorption capacity of the remaining dye in each therapeutic solution. We looked at the influence of temperature, pH, and communication time. The quantity of adsorb ate retained (mg) based on the weight of the adsorbent (g) is known as the adsorption capacity. It is expressed as the ratio x/m.

| | |
|---|---|
| Elimination %=[($C_0$-Ce)/$C_0$]×100 | (1) |

Ce denotes the dye's residual concentration at equilibrium in milligrams per liter, while C0 is the dye's original concentration in milligrams per liter .[14]

## 3. RESULTS AND DISCUSSION EFFECT OF CONTACT TIME

It was investigated whether there was an association between the contact time and the percentage of azure C removed utilizing modified pomegranate peel powder., as shown in Figure.2. The subsequent information refers to the balance time where, concentration 5 mg/L dye concentration, and 0.01 g adsorbent weight were studied.

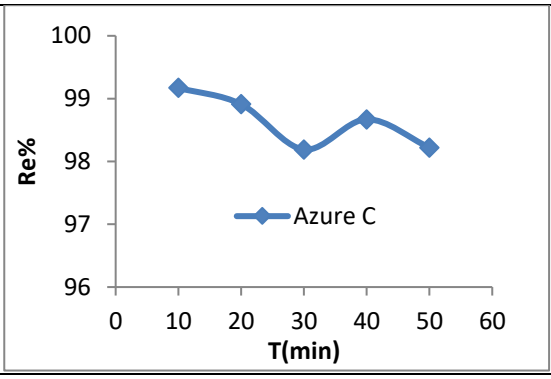


**Figure 2.** Effects of contact time on the adsorption of Azure C by modified pomegranate peel powder at 298 K

### Effect of adsorbent weight

The experiment was conducted at 298 K with an initial dye concentration of 5 mg/L and an adsorbent weight range of 0.01~0.05) g. As weight increases, Figure 3 illustrates that removal efficiency rises as well. The increase in surface area of the adsorbent (modified pomegranate peel powder) results in an increase in its adsorption capacity. The pomegranate peel powder's active sites are then saturated when the efficiency reaches a constant value. Decide on 0.01 g as the ideal adsorbent material weight.

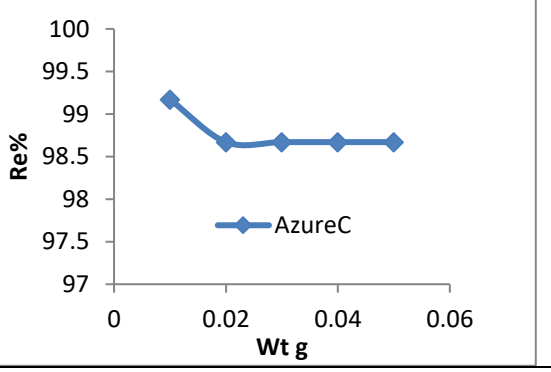


**Figure 3.** Effect of adsorbent weight on the adsorption of Azure C by modified pomegranate peel powder.

### Effect of pH

Two, four, six, eight, and ten pH values were selected in order to comprehend the impact of acidity

on the clearance ratio. The investigated dye was generated at a concentration of 5 mg/L, and its pH was adjusted to fall within the stated pH range by adding 0.01N of HCL and NaOH. Thus, in the other trials, pH 8 was used.

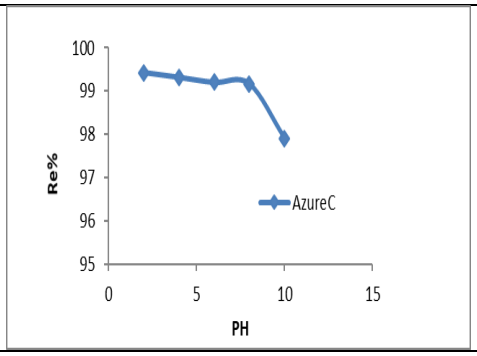


**Figure 4.** At 298K, the bearing of pH on the modified pomegranate peel powder's adsorption of Azure C was examined.

## Effect of Temperature

By examining the impacts of temperature variations, the study sought to understand the nature of the adsorption process. According to their respective formulae, Table 2 displays the computed values of the adsorption parameters, such as Gibbs energy (ΔG), enthalpy (ΔH), and entropy (ΔS). [15,16] .

| | |
|---|---|
| $\Delta G = - R T \ln K_{eq}$ | (2) |
| $\Delta S = (\Delta H - \Delta G) / T$ | (3) |

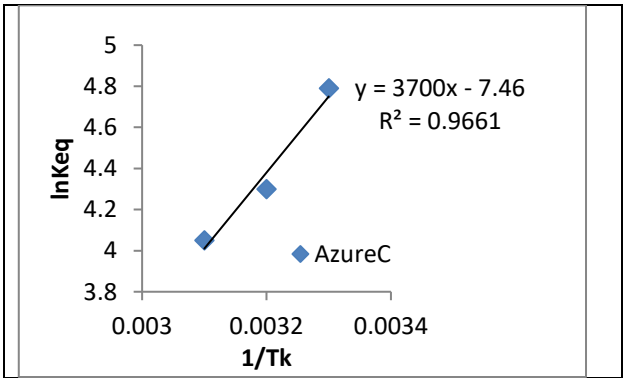


**Figure 5.** The effect of temperature on the modified pomegranate peel powder's adsorption of blue C dye.

**TABLE 1.** lists the temperature range in which the thermodynamic parameters ΔG, ΔH, and ΔS of the Azure C dye adsorbed on the modified surface of pomegranate peel powder were measured: 298 K to 318 K

| (Adsorbate) | Temp. K | - ΔG (KJ/mol) | - ΔH (KJ/mol) | - ΔS (KJ/mol.K) |
|---|---|---|---|---|
| **Azure C** | **298** | 11.86 | 30.76 | 0.063 |
| | **308** | 11.01 | | 0.064 |
| | **318** | 10.70 | | 0.063 |

We note that the functions "ΔG" and "ΔH" have negative values based on the values provided before. This implies an exothermic, spontaneous adsorption mechanism. Furthermore, during the adsorption process, the indications of ΔS show a reduction in the randomness of the adsorbed molecules. [17,18].

## Adsorption Isotherm

The adsorption isotherms of Azure C on the modified pomegranate peel powder were depicted in Figure 6. The experimental conditions included pH 8.0, temperatures ranging from 298 K to 318 K, 0.01 g of adsorbent material, dye concentrations ranging from 1 mg/L to 9 mg/L, and a contact time of 10 minutes. These figures provide an evidence that the adsorption capacity increases with higher equilibrium concentrations of Azure C.

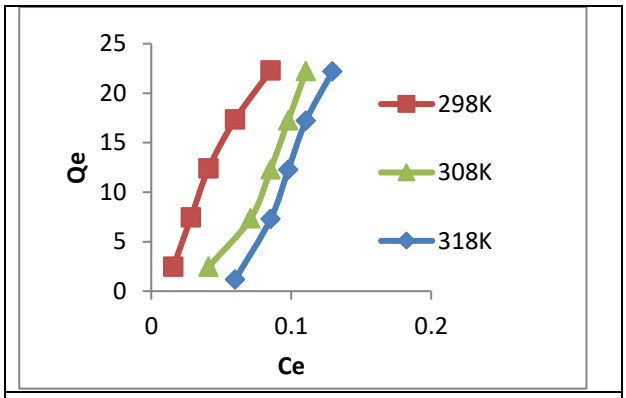


**Figure 6.** shows the adsorption isotherms of an aqueous solution containing Azure-C at different temperatures on modified pomegranate peel powder.

## Langmuir isotherm

Based on the homogenous sites of the adsorbent, the Langmuir isotherm postulates that the adsorption process occurs[19].

| | |
|---|---|
| $1/ab + Ce/a = Ce/Qe$ | (4) |

Where:

Ce is the equilibrium concentration of Azure C dye in the solution (mg/L), whereas Qe is the amount of Azure C dye that has been adsorbed at the equilibrium point (mg/g). A and B stand for the Langmuir constants. We can see the isotherm in Figure 7.

The connection between the Langmuir constant (b), the initial dye concentration in the solution (C0), and the Separation Factor (RL) is seen in equation (7). In particular:

$$R_L \text{ is } = 1 / (1 + b * Co) \quad (5)$$

Where:

The starting dye concentration in the solution is indicated by $C_0$ (mg/L).

Langmuir constant (L/mg) is denoted by b.

The Separation Factor is $R_L$.

The $R_L$ values show the kind of adsorption.is indicated by an $R_L$ value of 0, favorable adsorption Irreversible adsorption is shown by an $R_L$ value between 0 and 1, linear adsorption is indicated by an $R_L$ value of 1, and unfavorable adsorption is specified by an $R_L$ value larger than 1. [ 20,21]

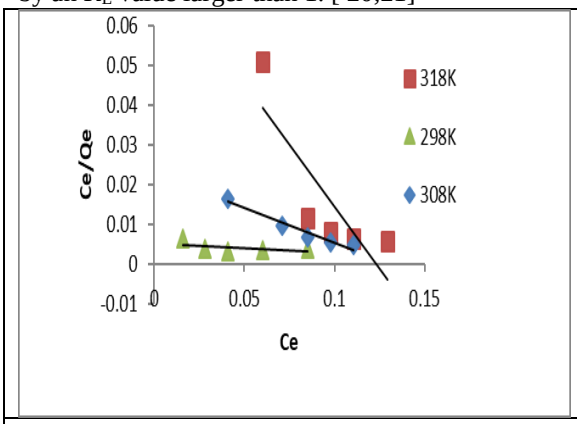


**Figure 7.** Using Langmuir isotherms, the adsorption of Azure C dye was investigated at different temperatures.

## Freundlich isotherm

Log Ce and log Qe have a connection which described by the Freundlich isotherm, an empirical equation based on adsorption on a heterogeneous surface [22]. Figure 8 shows how these relationships work. The Freundlich isotherm may be expressed using this equation.

$$\log Kf + 1/n \log Ye = \log Qe \quad (6)$$

Where, Qe is the quantity adsorbed at equilibrium (mg/g), Ce is the adsorbate's equilibrium concentration (Azure C), Kf is the adsorption capacity, and n is the adsorption strength.An empirical equation based on adsorption on a heterogeneous surface is called the Freundlich model [23,24].

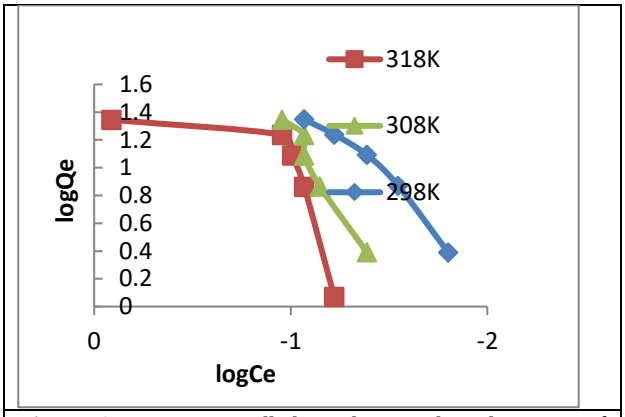


**Figure 8.** Using Freundlich isotherms, the adsorption of Azure C dye was examined at various temperatures.

**TABLE 2.** shows the analysis of Azure C's adsorption isotherms at temperatures between 298 and 318 K to find the Langmuir and Freundlich parameters.

| | adsorbate azure C | | | | | | |
|---|---|---|---|---|---|---|---|
| **T** | **langmuir isotherms** | | | | **freundlich isotherms** | | |
| | **a(mg/g)** | **b(mg/L)** | **$r^2$** | **$R_L$** | **Kf** | **n** | **$r^2$** |
| 2 | - 36.496 | - 0.0806 | 0.3397 | 1.675 | 663.132 | 0.7663 | 0.9582 |
| 3 | -5.787 | - 7.6123 | 0.95 | - 0.0269 | 3756.64 | 0.4348 | 0.9639 |
| 3 | -1.603 | - 0.1227 | 0.7104 | 0.2279 | 36.5510 | 1.3518 | 0.4233 |

## 4. CONCLUSION

This work shows that modified pomegranate peel powder works well as an adsorbent to remove Azure-C dye from aqueous solutions. The results provide more advantages for treating industrial wastewater than just cutting waste. The adsorption of blue C dye on the surface of pomegranate peel powder is a spontaneous isothermal process, as demonstrated by the thermodynamic function.

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## Arabic Abstract

تم استخدام سطح الرمان المحورة والذي أظهرت نتائج جيدة وكفاءة عالية في إزالة الملوثات من المحاليل المائية. تم استعمال السطح لإزالة صبغة C الزرقاء من محاليلها المائية والتي تعتبر من الصبغات الضارة. وتمت دراسة الظروف التالية (زمن التوازن، الوزن، دالة الرقم الهيدروجيني، درجة الحرارة)، وبلغت نسبة الإزالة 99.17% عند زمن 10 دقائق، وزن، درجة حرارة25مئوية، وPH 8 ، والثوابت الديناميكية الحرارية (ΔS، ΔH، ΔG ) وتمت دراسة ازوثيرمات الحرارة (Freundlich ،Langmuir).

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# A statistical study of the high prolactin hormone and its relationship to male and female infertility in the Najaf district

Nada Habeeb Obaid [1]

[1] *College of Pure Science, Dep. of Chemistry, University of Kerbala, Iraq*

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Aim of study: Toward studying the effects of prolactin, LH, and FSH hormones on male infertility in Najaf district.

Methods: One hundred and sixty-four healthy, fertile men and women served as the control group, whereas 184 infertile men and women from enrollments came from the Infertility Center, Al-Sadr Medical City, and Al. Najaf City. They are in the age range of 20 to 70.

Results: According to the findings, the LH mean ± st the control group's IU/L was 4.37±0.35. There were notable variations in the F.S.H. value (8.74±1.43IU/L).

andard deviation was 16.25±4.6 IU/L, whereas the value for the control group was 4.37±0.35 IU/L. Significant differences were seen in the F.S.H. value (8.74±1.43IU/L) between the infertile patient group and the control group (men and women) (5.33±0.44IU/L).,

Prolactin levels in the patients were 53.12±5.68 (IU/L) and 19.25±1.23 (IU/L) in the control group, respectively. There was no statistically significant difference between the control group and the cases study. The same can be said about control group or the cases study.On the other hand, levels of prolactin and F.S.H. fall modestly with age, whereas levels of L.H. increase.

Conclusion: The findings of this study indicate that in order to classify infertility patients into male and female groups and estimate a comprehensive case and control, research should be carried out for both male and female infertility due to the hormonal imbalance of L.H., F.S.H., and prolactin.

## 1. INTRODUCTION

Prolactin is a polypeptide hormone that is responsible for lactation in mammals, breast growth, and a variety of other functions. Prolactin is made up of 199 amino acids, followed by proteolytic cleavage of the signal peptide from the prolactin prohormone (pre-prolactin), and finally post-translational modification. Prolactin is secreted from the anterior part of the pituitary gland and its secretion is under the control and regulation of the hypothalamus. The immunological system, the uterus, and the mammary glands, in addition to the central nervous system, are all capable of producing prolactin [1]. Prolactin has regulating properties. Prolactin is generated by a number of different organs, although its expression is highest in the pituitary gland [2].

*Corresponding Author Institutional Email:
nada.h@uokerbala.edu.iq (Nada Habeeb Obaid)

vertebrates, but it was already largely expressed in specific cells in the pituitary in fish, and prolactin was released into the bloodstream as a multifunctional hormone. That is a class of cytokines based on the structure and type of receptor prolactin. Pituitary hormones consist of prolactin and growth hormone. The structure of prolactin and growth hormones differs from that of other pituitary hormones. Prolactin is also uncommon among adenohypophysis hormones in that it does not have an endocrine gland target, facilitate its functions, but it now acts directly through prolactin receptors found in a number of target organs [3]. The majority of prolactin's primary targets are epithelial cells, on which it can have proliferative effects as well as faster gene manifestation and even faster molecular actions [4].

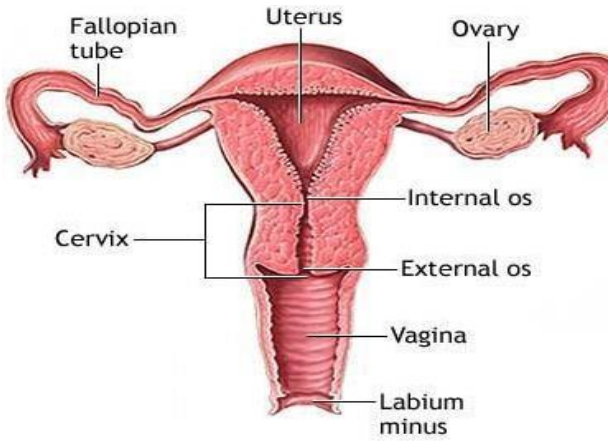


**Figure1.** Explained the reproductive system of women [1].

Breastfeeding and the growth of breasts are regulated by the polypeptide hormone prolactin. Proteolytic enzymes cleave the prolactin prohormone's signal peptide, or pre-prolactin, and post-translational modification occurs next. 199 amino acids make up prolactin. The anterior pituitary gland region produces prolactin, which the brain regulates and controls. In addition to the central nervous system, other organs that can produce prolactin include the uterus, mammary glands, and the immune system [3]. The hormone prolactin is characterized by its regulating properties. Although it is produced by many organs, the pituitary gland produces the most of it [4].

Prolactin was widely synthesized in specific cells in the fish pituitary before it was released into the bloodstream as a multifunctional hormone in vertebrates. Prolactin receptor type and shape indicate that this is a class of cytokines. The pituitary produces two hormones: growth hormone and prolactin. Prolactin and growth hormones are not the same structure as other pituitary hormones. Prolactin is distinct from other adenohypophysis hormones in that it now functions directly through prolactin receptors found in a range of target organs, rather than relying on an endocrine gland target to assist it in its duties [5]. Prolactin primarily acts on epithelial cells, where it can promote proliferation and accelerate gene expression and molecular activity [6]. Complicated medical conditions have affected people worldwide, exacerbating psychosocial and economic problems. In addition, it's believed that between 60 and 80 million couples worldwide struggle with infertility annually [7]. The pituitary glands in both sexes generate L.H., a hormone associated with women. Together with other hormones, it regulates the menstrual cycle F.S.H ) [8] .

At higher than normal levels of (L.H) in females may indicate the following: the pituitary gland is malfunctioning, the ovaries are absent or not functioning, Ovulation, or the release of the egg from the ovary is taking place, and both the male and female hypothalamus are under stress [9], wellness that promotes productivity in both men and women. Furthermore, there is a connection between men's and women's sexual satisfaction and Prolactin Stimulates Lactation (PSL) [10].

## 2. MATERIALS and METHODS

This research comprised 164 infertile patients, both male and female. The patients were recruited from the infertility center in October 2020 to April 2021 in Al-Sadr Medical City and Najaf City. They range in age from twenty to seventy. The control group included one hundred people in good health,both male and female, whose ages matched those of infertile couples and who showed no outward symptoms of disease.

### 2.1. Hormonal Test

All three hormones under study—prolactin, L.H., and F.S.H.—were measured utilizing the Bio.Tek. Instruments 217337, U.S.A. Enzyme-Linked Immune Sorbent Assay (ELISA) apparatus. It takes prolactin for them to be able to produce.

### 2.2. The Statistical Examination

The Standard Error (SE) represents the average value of the data. The Student t-test and Pearson's correlation coefficients in the concentrations of L.H., F.S.H., and Prolactin were used to evaluate the data [11].

## 3. RESULTS AND DISCUSSIONS

Infertility is a public reproductive disorder that is marked by a couple's inability to conceive after at least a year of frequent, unprotected sexual contact. Stress affects men and women equally, and chronic stress can result in major health problems, depression, and anxiety disorders [12]. An essential component of their capacity to procreate is (prolactin). Hormonal disorders that affect ovulation include hyperprolactinemia, hypoprolactinemia, and hyperthyroidism. Recently, the diagnosis and causes of infertility have been studied with a focus on hormonal imbalances. Furthermore, an increase in the proportion of females (F.S.H.) may indicate a decline in the number of embryos and Good Quality Eggs (GQE) created for fertilization. Depending on her age, a woman may have fewer odds of becoming pregnant than she would have thought. Pregnancy is not difficult to achieve in spite of this. She might struggle to conceive and need infertility therapy [13].

Table 1 shows that Scott et al. (1989) and Ban et al. (2013) concur that infertility is associated with a hormonal imbalance. Numerous anthropometric and socioeconomic parameters have been related to fecundity.Therefore, an assessment of the prolactin, LH, and FSH levels in infertility was conducted.It had greater levels of FSH, LH,

and prolactin. These results are in line with research conducted in 2013 by Ban *et al*. and Aroma *et al*. Higher levels of the hormone prolactin are often linked to infertility rather than fertility, according to studies by Scott MG *et al.* (1989) and Choudhury *et al.* (1995) [14].

**TABLE 1.** Age, FSH, LH, and prolactin levels are the outcomes of infertility in both men and women.

| | type | Cases(164) | Mean ± SE | P-value |
|---|---|---|---|---|
| Age | male | 42 | 38.48±2.27 | 0.486 |
| | female | 121 | 36.67±1.31 | |
| F.S.H I.U/L | male | 42 | 3.43±0.81 | 0.105 |
| | female | 121 | 5.31±0.62 | |
| L.H I.U/L | male | 42 | 8.66± 2.90 | 0.681 |
| | female | 121 | 7.59±1.16 | |
| Prolactin. | male | 42 | 22.06± 2.49 | 0.112 |
| I.U/L | female | 121 | 31.06± 3.19 | |

Table (2) displays the age and hormone levels for the control group (male and female) and the investigative cases (female and male infertility). For LH, the value was 16. 25±4.06 IU/L, while for the control group, it was 4.37±0.35).The (F.S.H.) value for infertility cases (8.741.43) IU/L differed statistically from the value of the male and female control group. The case group had prolactin levels of 19.251.23 IU/L and the control group had 53.125.68 IU/L. The case study and control groups did not show any appreciable differences.

**TABLE 2.** Comparing hormonal instances (prolactin, L.H., and F.S.H.) with hormonal control.

| | CASES AVAR± SDNO(164) | CONTROL AVAR± SD NO(100) |
|---|---|---|
| L.H I.U/L | 16.25±4.o6 | 4.37±0.35 |
| F.S,H I.U/L | 8.74±1.43 | 5.33±0.44 |
| Prolactin I.U/L | 53.12±5.68 | 19.25±1.23 |

Furthermore, utilizing Pearson's correlation coefficient, the findings demonstrated no significant differences in the concentrations of the other research parameters (F.S.H. and Prolactin), and a positive significant link between age and L.H. (r= 0.478, p 000). table (3).

This study contradicts recent reports by Dabbous and Atkin (2018) that hyperprolactinemia causes excessively high quantities of adrenal steroids to be synthesized and secreted.[4] In study covering both male and female participants, hormonal imbalance—specifically, L.H., F.S.H., and Prolactin—is merely one potential contributing cause to infertility.

The levels of F.S.H. rise with age, whereas the levels of L.H. and Prolactin somewhat fall with age, as shown in table (3).

**TABLE 3.** Correlation between the studied infertility parameter for men and women.

| Parameters N= 146 | Parameter | r | P-value |
|---|---|---|---|
| | F.S.H. IU/L | 0.46 | 0.12 |
| Age | L.H. IU/L | 0..478** | 0.000 |
| | Prolactin. IU/L | 0.052 | 0.11 |

**** the correlation's significance level of 0.01.**

Prolactin is a protein hormone of the anterior pituitary gland that was originally named for its ability to promote lactation in response to the suckling stimulus of hungry young mammals. [15] Essentially, the pituitary gland is the primary source of almost all of the prolactin in normal individuals. A dditional pituitary and prolactin glands may also contribute significantly. However, they have different functions and mainly influence the surrounding environment by means of autocrine and paracrine processes [16]. This study disagrees with Langer, *et al*, (1991) study whose results were independent of prolactin (HPRL) levels or amenorrhea. [17-18].

The present study concludes that the findings contradict the claims made by Yu-lee (1997) and Bachelot (2007). They state that hyperprolactinemia is frequently linked to infertility in both males and females and it is caused by abnormal sexual and reproductive roles or agalactorrhea [19-20-21].

Data on falling fertility and infertile age indicate that pregnancy rates decrease gradually in the early 30s but significantly in the late 30s and early 40s [2].

## 3. CONCLUSTION

This is a conclusion, in order to determine the most likely investigative hormonal tests in the predictable work of infertility clinics, a thorough case-control study is carried out to estimate hormonal imbalance (L.H., F.S.H., and Prolactin) hormones in male and female infertility. The levels of hyperprolactinemia in the case study and control groups did not differ substantially from one another.

Additionally, using Pearson's correlation coefficient, the results showed a significant positive correlation between age and (L.H.), with no appreciable differences in the concentrations of the other parameters (F.S.H. and Prolactin) that were being studied.

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## Arabic Abstract


**هدف الدراسة:** نحو دراسة تأثير هرمونات البرولاكتين و LH و FSH على العقم عند الذكور في منطقة النجف الأشرف.

**الطرق:** مائة وأربعة وستون من الرجال والنساء الأصحاء والخصبين كانوا بمثابة المجموعة الضابطة، في حين أن 184 رجلاً وامرأة يعانون من العقم من المسجلين جاءوا من مركز العقم ومدينة الصدر الطبية وآل. مدينة النجف. تتراوح أعمارهم بين 20 إلى 70 عامًا.

**النتائج:** وفقا للنتائج، كان متوسط LH ± st IU / L للمجموعة الضابطة 4.37 ± 0.35. كانت هناك اختلافات ملحوظة في .F.S.H.القيمة (8.74 ± 1.43 وحدة دولية / لتر).كان الانحراف المعياري 16.25 ± 4.6 وحدة دولية / لتر، في حين كانت القيمة لمجموعة التحكم 4.37 ± 0.35 وحدة دولية / لتر. شوهدت اختلافات كبيرة في .F.S.H.القيمة (8.74 ± 1.43 وحدة دولية / لتر) بين مجموعة المرضى الذين يعانون من العقم والمجموعة الضابطة (الرجال والنساء) (5.33 ± 0.44 وحدة دولية / لتر).،

كانت مستويات البرولاكتين في المرضى 53.12 ± 5.68 (وحدة دولية / لتر) و 19.25 ± 1.23 (وحدة دولية / لتر) في المجموعة الضابطة، على التوالي. لم يكن هناك فروق ذات دلالة إحصائية بين المجموعة الضابطة ودراسة الحالات، ولا المجموعة الضابطة أو دراسة الحالات.من ناحية أخرى، كانت مستويات البرولاكتين وهرمون .F.S.H.تنخفض بشكل طفيف مع التقدم في السن، بينما ترتفع مستويات .L.H

**الخلاصة**: تشير نتائج هذه الدراسة إلى أنه من أجل تصنيف مرضى العقم إلى مجموعات من الذكور والإناث وتقدير حالة شاملة وأبحاث مراقبة يجب إجراء أبحاث لكل من العقم عند الذكور والإناث بسبب عدم التوازن الهرموني .L.H، F.S.H، و. البرولاكتين.

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# Using Triple Simple Elliptic Absolute Orlicz Function Defined by Triple Sequences Spaces $(\ell_\infty)^3_{\mathbb{F}}(\Theta)$ with Fuzzy Metric

Aqeel Mohammed Hussein [1]

[1] *University of Al-Qadisiyah, Faculty of Education, Department of Mathematics*

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*A B S T R A C T*

We present the triple simple elliptic absolute Orlicz function in this paper, which is determined by triple sequence spaces with fuzzy metrics. We also discuss some of its properties, such as that the space $(\ell_\infty)^3_{\mathbb{F}}(\Theta)$ is symmetric, solid and complete .

## 1. INTRODUCTION

L.A. Zadeh introduced fuzzy set theory in 1965, and a number of scholars have since adopted it, including Yu-ru [10], Tripathy and Baruah [1], Tripathy and Borgohain [3], Tripathy and Dutta [4], Tripathy and Sarma [7], [8], [9], and many more.

Kramosil and Michalek [6] created the fuzzy metric space by extending the idea of the probabilistic metric space to the fuzzy scene.

The space $(\ell_\infty)^3_{\mathbb{F}}(\Theta)$ produced by the basic elliptic absolute Orlicz function with fuzzy metric is defined and introduced in this study.

$\vartheta : [0,\infty) \to [0,\infty)$ is called an Orlicz function; it is a continuous, non-decreasing and convex function with $\vartheta(0) = 0, \vartheta(\mathfrak{A}) \succ 0$ as $\mathfrak{A} \succ 0$ and $\vartheta(\mathfrak{A}) \to \infty$.

## 2. DEFINITIONS AND PRELIMINARIES

A simple elliptic absolute Orlicz function is a function $\mathbb{M} : [0,\infty) \to [0,\infty) \ni \mathbb{M}(\mathfrak{A}) = -|\mathfrak{A}|^2\vartheta(\mathfrak{A})$, where $\vartheta$ is an Orlicz function.

A triple simple elliptic absolute Orlicz function is a function $\Theta : [0,\infty) \times [0,\infty) \times [0,\infty) \longrightarrow [0,\infty) \times [0,\infty) \times [0,\infty) \ni \Theta(\mathfrak{A},\mathfrak{S},\mathfrak{R}) = \big(\Theta_1(\mathfrak{A}), \Theta_2(\mathfrak{S}), \Theta_3(\mathfrak{R})\big)$, where $\Theta_1 : [0,\infty) \to [0,\infty) \ni \Theta_1(\mathfrak{A}) = -|\mathfrak{A}|^2\vartheta_1(\mathfrak{A})$ , $\Theta_2 : [0,\infty) \to [0,\infty) \ni \Theta_2(\mathfrak{S}) = -|\mathfrak{S}|^2\vartheta_2(\mathfrak{S})$ , $\Theta_3 : [0,\infty) \to [0,\infty) \ni \Theta_3(\mathfrak{R}) = -|\mathfrak{R}|^2\vartheta_3(\mathfrak{R})$ . These functions are even, convex, continuous and non-decreasing , that hold the following conditions :

i) $\Theta_1(0) = 0, \Theta_2(0) = 0, \Theta_3(0) = 0 \Longrightarrow \Theta(0,0,0) = \big(\Theta_1(0), \Theta_2(0), \Theta_3(0)\big) = (0,0,0)$.

ii) $\Theta_1(\mathfrak{A}) \succ 0 , \Theta_2(\mathfrak{S}) \succ 0 , \Theta_3(\mathfrak{R}) \succ 0 \Longrightarrow \Theta(\mathfrak{A},\mathfrak{S},\mathfrak{R}) = \big(\Theta_1(\mathfrak{A}), \Theta_2(\mathfrak{S}), \Theta_3(\mathfrak{R})\big) \succ (0,0,0)$, for $\mathfrak{A} \succ 0, \mathfrak{S} \succ 0, \mathfrak{R} \succ 0$,by which we say $(\mathfrak{A},\mathfrak{S},\mathfrak{R}) \succ (0,0,0)$ as $\Theta_1(\mathfrak{A}) \succ 0 , \Theta_2(\mathfrak{S}) \succ 0 , \Theta_3(\mathfrak{R}) \succ 0$.

iii) $\Theta_1(\mathfrak{A}) \to \infty, \Theta_2(\mathfrak{S}) \to \infty, \Theta_3(\mathfrak{R}) \to \infty$ as $\mathfrak{A} \to \infty, \mathfrak{S} \to \infty, \mathfrak{R} \to \infty \Longrightarrow \Theta(\mathfrak{A},\mathfrak{S},\mathfrak{R}) = \big(\Theta_1(\mathfrak{A}), \Theta_2(\mathfrak{S}), \Theta_3(\mathfrak{R})\big) \to (\infty,\infty,\infty)$ as $(\mathfrak{A},\mathfrak{S},\mathfrak{R}) \to (\infty,\infty,\infty)$ by which we say $\Theta(\mathfrak{A},\mathfrak{S},\mathfrak{R}) \to (\infty,\infty,\infty)$ as $\Theta_1(\mathfrak{A}) \to \infty, \Theta_2(\mathfrak{S}) \to \infty, \Theta_3(\mathfrak{R}) \to \infty$ .

$\big(\ltimes_{\ell k j i}\, \mathfrak{A}_{\ell k j}\big) \in \mathbb{E}^3$ when $\big(\mathfrak{A}_{\ell k j}\big) \in \mathbb{E}^3$ for every sequence of scalars with $\big|\ltimes_{\ell k j}\big| \leqslant 1, \forall\, \ell, k, j \in \mathbb{N}$ implies that triple sequence space $\mathbb{E}^3$ is solid .

$\big(\mathfrak{A}_{\pi(\ell k j)}\big) \in \mathbb{E}^3$ when $\big(\mathfrak{A}_{\ell k j i}\big) \in \mathbb{E}^3$ leads to $\mathbb{E}^3$ is symmetric and $\pi$ is a permutation of $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$ .

*Corresponding Author Institutional Email: aqeel.Hussein@qu.edu.iq (Aqeel Mohammed Hussein)

$\mathcal{T} : \mathbb{R}(\mathbb{I}) \times \mathbb{R}(\mathbb{I}) \to \mathbb{R} \ni \mathcal{T}(\mathfrak{H}, \mathfrak{G}) = \sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa(\mathfrak{H}^\kappa, \mathfrak{G}^\kappa), \mathcal{T}_\kappa : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \ni \mathcal{T}_\kappa(\mathfrak{H}^\kappa, \mathfrak{G}^\kappa) = \min\{|\mathfrak{H}_1^\kappa - \mathfrak{G}_1^\kappa|, |\mathfrak{H}_2^\kappa - \mathfrak{G}_2^\kappa|\}$ and $\mathcal{S} : \mathbb{R}(\mathbb{I}) \times \mathbb{R}(\mathbb{I}) \to \mathbb{R} \ni \mathcal{S}(\mathfrak{H}, \mathfrak{G}) = \sup_{0<\kappa\leqslant 1} \mathcal{S}_\kappa(\mathfrak{H}^\kappa, \mathfrak{G}^\kappa), \mathcal{S}_\kappa : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \ni \mathcal{S}_\kappa(\mathfrak{H}^\kappa, \mathfrak{G}^\kappa) = \max\{|\mathfrak{H}_1^\kappa - \mathfrak{G}_1^\kappa|, |\mathfrak{H}_2^\kappa - \mathfrak{G}_2^\kappa|\}$

A Fuzzy metric space is the quadruple $(\mathbb{R}(\mathbb{I}), d_{\mathbb{F}}, \mathbb{T}, \mathbb{S})$, and $d_{\mathbb{F}}$ is a fuzzy metric if :

i) $d_{\mathbb{F}}(\mathfrak{E}, \mathfrak{C}) = 0 \Leftrightarrow \mathfrak{E} = \mathfrak{C}, \forall\, \mathfrak{E}, \mathfrak{C} \in \mathbb{R}(\mathbb{I})$.

ii) $d_{\mathbb{F}}(\mathfrak{E}, \mathfrak{C}) = d_{\mathbb{F}}(\mathfrak{C}, \mathfrak{E}), \forall\, \mathfrak{E}, \mathfrak{C} \in \mathbb{R}(\mathbb{I})$.

iii) $\forall\, \mathfrak{E}, \mathfrak{C}, \mathfrak{G} \in \mathbb{R}(\mathbb{I})$,

a) $d_{\mathbb{F}}(\mathfrak{E}, \mathfrak{C})(\mathfrak{Q} + \mathfrak{O}) \geqslant \mathbb{T}\big(d_{\mathbb{F}}(\mathfrak{E}, \mathfrak{G})(\mathfrak{Q}), d_{\mathbb{F}}(\mathfrak{G}, \mathfrak{C})(\mathfrak{O})\big)$, whenever $\mathfrak{Q} \leqslant \mathcal{T}_1(\mathfrak{E}, \mathfrak{G}), \mathfrak{O} \leqslant \mathcal{T}_1(\mathfrak{G}, \mathfrak{C})$ and $\mathfrak{Q} + \mathfrak{O} \leqslant \mathcal{T}_1(\mathfrak{E}, \mathfrak{C})$.

b) $d_{\mathbb{F}}(\mathfrak{E}, \mathfrak{C})(\mathfrak{Q} + \mathfrak{O}) \leqslant \mathbb{T}\big(d_{\mathbb{F}}(\mathfrak{E}, \mathfrak{G})(\mathfrak{Q}), d_{\mathbb{F}}(\mathfrak{G}, \mathfrak{C})(\mathfrak{O})\big)$, whenever $\mathfrak{Q} \geqslant \mathcal{T}_1(\mathfrak{E}, \mathfrak{G}), \mathfrak{O} \geqslant \mathcal{T}_1(\mathfrak{G}, \mathfrak{C})$ and $\mathfrak{Q} + \mathfrak{O} \geqslant \mathcal{T}_1(\mathfrak{E}, \mathfrak{C})$.

$(\ell_\infty)^3_{\mathbb{F}}(\Theta) = \Big\{\mathfrak{X}_{abc} = ((\mathfrak{X}_1)_{abc}, (\mathfrak{X}_2)_{abc}, (\mathfrak{X}_3)_{abc}) \in \mathbb{W}^3_{\mathbb{F}} : \sup_{abc}\Big[\Theta_1\Big(\frac{\mathcal{T}((\mathfrak{X}_1)_{abc}, \bar{0})}{\rho}\Big) \curlyvee \Theta_2\Big(\frac{\mathcal{T}((\mathfrak{X}_2)_{abc}, \bar{0})}{\rho}\Big) \curlyvee \Theta_3\Big(\frac{\mathcal{T}((\mathfrak{X}_3)_{abc}, \bar{0})}{\rho}\Big)\Big] \prec (\infty, \infty, \infty)$ and $\sup_{abc}\Big[\Theta_1\Big(\frac{\mathcal{S}((\mathfrak{X}_1)_{abc}, \bar{0})}{\rho}\Big) \curlyvee \Theta_2\Big(\frac{\mathcal{S}((\mathfrak{X}_2)_{abc}, \bar{0})}{\rho}\Big) \curlyvee \Theta_3\Big(\frac{\mathcal{S}((\mathfrak{X}_3)_{abc}, \bar{0})}{\rho}\Big)\Big] \prec (\infty, \infty, \infty)$, for some $\rho \succ 0\Big\}$, where $\Theta = (\Theta_1, \Theta_2, \Theta_3)$.

## 3. MAIN RESULTS

**Theorem 3.1:**

$(\ell_\infty)^3_{\mathbb{F}}(\Theta)$ is metric space under the metric :

$\bar{d}(\mathfrak{A}, \mathfrak{S})_\Theta = \inf\Big\{(\rho, \rho, \rho) \succ (0,0,0) : \sup_{abc}\Big[\Theta_1\Big(\frac{\mathcal{T}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc})}{\rho}\Big) \curlyvee \Theta_2\Big(\frac{\mathcal{T}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc})}{\rho}\Big) \curlyvee \Theta_3\Big(\frac{\mathcal{T}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc})}{\rho}\Big)\Big] \leqslant (1,1,1)$ and $\sup_{abc}\Big[\Theta_1\Big(\frac{\mathcal{S}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc})}{\rho}\Big) \curlyvee \Theta_2\Big(\frac{\mathcal{S}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc})}{\rho}\Big) \curlyvee \Theta_3\Big(\frac{\mathcal{S}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc})}{\rho}\Big)\Big] \leqslant (1,1,1)\Big\}, \forall\, \mathfrak{A} = (\mathfrak{A}_1, \mathfrak{A}_2, \mathfrak{A}_3), \mathfrak{S} = (\mathfrak{S}_1, \mathfrak{S}_2, \mathfrak{S}_3) \in (\ell_\infty)^3_{\mathbb{F}}(\Theta)$ .

**Proof**

$\forall\, \mathfrak{A}, \mathfrak{S} \in (\ell_\infty)^3_{\mathbb{F}}(\Theta)$, we get,

i) If $\bar{d}(\mathfrak{A}, \mathfrak{S})_\Theta = 0$.

This implies that

$\mathcal{T}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc}) = 0, \mathcal{T}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc}) = 0, \mathcal{T}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc}) = 0,$

and

$\mathcal{S}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc}) = 0, \mathcal{S}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc}) = 0, \mathcal{S}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc}) = 0,$

( since $\Theta_1(0) = 0, \Theta_2(0) = 0, \Theta_3(0) = 0$).

This indicate that , for all $\kappa \in (0,1]$ ,

$\mathcal{T}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc}) = \sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa((\mathfrak{A}_1)^\kappa_{abc}, (\mathfrak{S}_1)^\kappa_{abc}) = 0 \Rightarrow \mathcal{T}_\kappa((\mathfrak{A}_1)^\kappa_{abc}, (\mathfrak{S}_1)^\kappa_{abc}) = 0, \forall\, \kappa \in (0,1],$

and

$\Rightarrow \min\{|(\mathfrak{A}_1)^\kappa_{abc1} - (\mathfrak{S}_1)^\kappa_{abc1}|, |(\mathfrak{A}_1)^\kappa_{abc2} - (\mathfrak{S}_1)^\kappa_{abc2}|\} = 0 \ldots\ldots (1)$

and

$\mathcal{T}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc}) = \sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa((\mathfrak{A}_2)^\kappa_{abc}, (\mathfrak{S}_2)^\kappa_{abc}) = 0 \Rightarrow \mathcal{T}_\kappa((\mathfrak{A}_2)^\kappa_{abc}, (\mathfrak{S}_2)^\kappa_{abc}) = 0, \text{for all } \kappa \in (0,1]$

$\Rightarrow \min\{|(\mathfrak{A}_2)^\kappa_{abc1} - (\mathfrak{S}_2)^\kappa_{abc1}|, |(\mathfrak{A}_2)^\kappa_{abc2} - (\mathfrak{S}_2)^\kappa_{abc2}|\} = 0, \ldots\ldots (2)$

and

$\mathcal{T}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc}) = \sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa((\mathfrak{A}_3)^\kappa_{abc}, (\mathfrak{S}_3)^\kappa_{abc}) = 0 \Rightarrow \mathcal{T}_\kappa((\mathfrak{A}_3)^\kappa_{abc}, (\mathfrak{S}_3)^\kappa_{abc}) = 0, \text{for all } \kappa \in (0,1]$

$\Rightarrow \min\{|(\mathfrak{A}_3)^\kappa_{abc1} - (\mathfrak{S}_3)^\kappa_{abc1}|, |(\mathfrak{A}_3)^\kappa_{abc2} - (\mathfrak{S}_3)^\kappa_{abc2}|\} = 0, \forall\, \kappa \in (0,1] \ldots\ldots (3)$

Similarly, for all $\kappa \in (0,1]$,

$\mathcal{S}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc}) = \sup_{0<\kappa\leqslant 1} \mathcal{S}_\kappa((\mathfrak{A}_1)^\kappa_{abc}, (\mathfrak{S}_1)^\kappa_{abc}) = 0 \Rightarrow \mathcal{S}_\kappa((\mathfrak{A}_1)^\kappa_{abc}, (\mathfrak{S}_1)^\kappa_{abc}) = 0, \text{for all } \kappa \in (0,1]$

and

$\Rightarrow \max\{|(\mathfrak{A}_1)^\kappa_{abc1} - (\mathfrak{S}_1)^\kappa_{abc1}|, |(\mathfrak{A}_1)^\kappa_{abc2} - (\mathfrak{S}_1)^\kappa_{abc2}|\} = 0 \ldots\ldots (4)$

and

$\mathcal{S}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc}) = \sup_{0<\kappa\leqslant 1} \mathcal{S}_\kappa((\mathfrak{A}_2)^\kappa_{abc}, (\mathfrak{S}_2)^\kappa_{abc}) = 0 \Rightarrow \mathcal{S}_\kappa((\mathfrak{A}_2)^\kappa_{abc}, (\mathfrak{S}_2)^\kappa_{abc}) = 0, \text{ for all } \kappa \in (0,1]$

$\Rightarrow \max\{|(\mathfrak{A}_2)^\kappa_{abc1} - (\mathfrak{S}_2)^\kappa_{abc1}|, |(\mathfrak{A}_2)^\kappa_{abc2} - (\mathfrak{S}_2)^\kappa_{abc2}|\} = 0 \ldots\ldots (5)$

and

$\mathcal{S}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc}) = \sup_{0<\kappa\leqslant 1} \mathcal{S}_\kappa((\mathfrak{A}_3)^\kappa_{abc}, (\mathfrak{S}_3)^\kappa_{abc}) = 0 \Rightarrow \mathcal{S}_\kappa((\mathfrak{A}_3)^\kappa_{abc}, (\mathfrak{S}_3)^\kappa_{abc}) = 0, \forall\, \kappa \in (0,1]$

$\Rightarrow \max\{|(\mathfrak{A}_3)^\kappa_{abc1} - (\mathfrak{S}_3)^\kappa_{abc1}|, |(\mathfrak{A}_3)^\kappa_{abc2} - (\mathfrak{S}_3)^\kappa_{abc2}|\} = 0, \text{for all } \kappa \in (0,1] \ldots\ldots (6)$

Based on (1), (2), (3), (4), (5), (6) , this indicates that , $(\mathfrak{A}_1)_{abc} = (\mathfrak{S}_1)_{abc}, (\mathfrak{A}_2)_{abc} = (\mathfrak{S}_2)_{abc}, (\mathfrak{A}_3)_{abc} = (\mathfrak{S}_3)_{abc} \Rightarrow \mathfrak{A} = \mathfrak{S}, \forall\, \mathfrak{a}, \mathfrak{b}, \mathfrak{c} \in \mathbb{N}$.

Conversely $\mathfrak{A} = \mathfrak{S}$.

Then, using $\mathcal{T}'s, \mathcal{S}'s$ definitions , we obtain

$\mathcal{T}_\kappa((\mathfrak{A}_1)^\kappa_{abc}, (\mathfrak{S}_1)^\kappa_{abc}) = 0, \mathcal{T}_\kappa((\mathfrak{A}_2)^\kappa_{abc}, (\mathfrak{S}_2)^\kappa_{abc}) = 0, \mathcal{T}_\kappa((\mathfrak{A}_3)^\kappa_{abc}, (\mathfrak{S}_3)^\kappa_{abc}) = 0,$

and

$\mathcal{S}_\kappa((\mathfrak{A}_1)^\kappa_{abc}, (\mathfrak{S}_1)^\kappa_{abc}) = 0, \mathcal{S}_\kappa((\mathfrak{A}_2)^\kappa_{abc}, (\mathfrak{S}_2)^\kappa_{abc}) = 0, \mathcal{S}_\kappa((\mathfrak{A}_3)^\kappa_{abc}, (\mathfrak{S}_3)^\kappa_{abc}) = 0, \forall\, \mathfrak{a}, \mathfrak{b}, \mathfrak{c} \in \mathbb{N}, \text{for all } \kappa \in (0,1].$

This means that,

$\sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa((\mathfrak{A}_1)^\kappa_{abc}, (\mathfrak{S}_1)^\kappa_{abc}) = 0, \sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa((\mathfrak{A}_2)^\kappa_{abc}, (\mathfrak{S}_2)^\kappa_{abc}) = 0, \sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa((\mathfrak{A}_3)^\kappa_{abc}, (\mathfrak{S}_3)^\kappa_{abc}) = 0$

and

$\sup_{0<\kappa\leqslant 1} \mathcal{S}_\kappa((\mathfrak{A}_1)^\kappa_{abc}, (\mathfrak{S}_1)^\kappa_{abc}) =$
$0, \sup_{0<\kappa\leqslant 1} \mathcal{S}_\kappa((\mathfrak{A}_2)^\kappa_{abc}, (\mathfrak{S}_2)^\kappa_{abc}) =$
$0, \sup_{0<\kappa\leqslant 1} \mathcal{S}_\kappa((\mathfrak{A}_3)^\kappa_{abc}, (\mathfrak{S}_3)^\kappa_{abc}) = 0, \forall\, \mathfrak{a}, \mathfrak{b}, \mathfrak{c} \in \mathbb{N}.$
Therefore $\mathcal{T}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc}) =$
$0, \mathcal{T}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc}) = 0, \mathcal{T}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc}) = 0$
and
$\mathcal{S}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc}) = 0, \mathcal{S}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc}) =$
$0, \mathcal{S}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc}) = 0$
Using $\Theta'$s continuity, we determine that $\bar{d}(\mathfrak{A}, \mathfrak{S})_\Theta = 0$ .
iii) $\bar{d}(\mathfrak{A}, \mathfrak{S})_\Theta = \inf\left\{(\rho, \rho, \rho) \succ (0,0,0) :\right.$
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc})}{\rho}\right) \curlyvee\right.$
$\left.\Theta_3\left(\frac{\mathcal{T}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc})}{\rho}\right)\right] \preccurlyeq$
$(1,1,1)$ and $\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc})}{\rho}\right) \curlyvee\right.$
$\left.\Theta_2\left(\frac{\mathcal{S}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{S}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc})}{\rho}\right)\right] \preccurlyeq$
$\left.(1,1,1)\right\}$.
Using $\mathcal{T}'$s definition, we have,
$\mathcal{T}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc}) = \sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa((\mathfrak{A}_1)^\kappa_{abc}, (\mathfrak{S}_1)^\kappa_{abc}) =$
$\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{A}_1)^\kappa_{abc1}, (\mathfrak{S}_1)^\kappa_{abc1}|, |(\mathfrak{A}_1)^\kappa_{abc2}, (\mathfrak{S}_1)^\kappa_{abc2}|\}) =$
$\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{S}_1)^\kappa_{abc1}, (\mathfrak{A}_1)^\kappa_{abc1}|, |(\mathfrak{S}_1)^\kappa_{abc2}, (\mathfrak{A}_1)^\kappa_{abc2}|\}) =$
$\sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa((\mathfrak{S}_1)^\kappa_{abc}, (\mathfrak{A}_1)^\kappa_{abc}) = \mathcal{T}((\mathfrak{S}_1)_{abc}, (\mathfrak{A}_1)_{abc})$,
and
$\mathcal{T}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc}) = \sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa((\mathfrak{A}_2)^\kappa_{abc}, (\mathfrak{S}_2)^\kappa_{abc}) =$
$\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{A}_2)^\kappa_{abc1}, (\mathfrak{S}_2)^\kappa_{abc1}|, |(\mathfrak{A}_2)^\kappa_{abc2}, (\mathfrak{S}_2)^\kappa_{abc2}|\})$ $\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{S}_2)^\kappa_{abc1}, (\mathfrak{A}_2)^\kappa_{abc1}|, |(\mathfrak{S}_2)^\kappa_{abc2}, (\mathfrak{A}_2)^\kappa_{abc2}|\}) =$
$\sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa((\mathfrak{S}_2)^\kappa_{abc}, (\mathfrak{A}_2)^\kappa_{abc}) = \mathcal{T}((\mathfrak{S}_2)_{abc}, (\mathfrak{A}_2)_{abc})$
and
$\mathcal{T}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc}) = \sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa((\mathfrak{A}_3)^\kappa_{abc}, (\mathfrak{S}_3)^\kappa_{abc}) =$
$\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{A}_3)^\kappa_{abc1}, (\mathfrak{S}_2)^\kappa_{abc1}|, |(\mathfrak{A}_3)^\kappa_{abc2}, (\mathfrak{S}_3)^\kappa_{abc2}|\}) =$
$\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{S}_3)^\kappa_{abc1}, (\mathfrak{A}_3)^\kappa_{abc1}|, |(\mathfrak{S}_3)^\kappa_{abc2}, (\mathfrak{A}_3)^\kappa_{abc2}|\}) =$
$\sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa((\mathfrak{S}_3)^\kappa_{abc}, (\mathfrak{A}_3)^\kappa_{abc}) = \mathcal{T}((\mathfrak{S}_3)_{abc}, (\mathfrak{A}_3)_{abc})$.
Therefore $\mathcal{T}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc}) =$
$\mathcal{T}((\mathfrak{S}_1)_{abc}, (\mathfrak{A}_1)_{abc}), \mathcal{T}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc}) =$
$\mathcal{T}((\mathfrak{S}_2)_{abc}, (\mathfrak{A}_2)_{abc}), \mathcal{T}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc}) =$
$\mathcal{T}((\mathfrak{S}_1)_{abc}, (\mathfrak{A}_1)_{abc})$ .
By proceeding in the same manner, we obtain,
$\mathcal{S}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc}) = \sup_{0<\kappa\leqslant 1} \mathcal{S}_\kappa((\mathfrak{A}_1)^\kappa_{abc}, (\mathfrak{S}_1)^\kappa_{abc}) =$
$\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{A}_1)^\kappa_{abc1}, (\mathfrak{S}_1)^\kappa_{abc1}|, |(\mathfrak{A}_1)^\kappa_{abc2}, (\mathfrak{S}_1)^\kappa_{abc2}|\}) =$
$\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{S}_1)^\kappa_{abc1}, (\mathfrak{A}_1)^\kappa_{abc1}|, |(\mathfrak{S}_1)^\kappa_{abc2}, (\mathfrak{A}_1)^\kappa_{abc2}|\}) =$
$\sup_{0<\kappa\leqslant 1} \mathcal{S}_\kappa((\mathfrak{S}_1)^\kappa_{abc}, (\mathfrak{A}_1)^\kappa_{abc}) = \mathcal{S}((\mathfrak{S}_1)_{abc}, (\mathfrak{A}_1)_{abc})$
and
$\mathcal{S}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc}) = \sup_{0<\kappa\leqslant 1} \mathcal{S}_\kappa((\mathfrak{A}_2)^\kappa_{abc}, (\mathfrak{S}_2)^\kappa_{abc}) =$
$\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{A}_2)^\kappa_{abc1}, (\mathfrak{S}_2)^\kappa_{abc1}|, |(\mathfrak{A}_2)^\kappa_{abc2}, (\mathfrak{S}_2)^\kappa_{abc2}|\}) =$
$\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{S}_2)^\kappa_{abc1}, (\mathfrak{A}_2)^\kappa_{abc1}|, |(\mathfrak{S}_2)^\kappa_{abc2}, (\mathfrak{A}_2)^\kappa_{abc2}|\}) =$
$\sup_{0<\kappa\leqslant 1} \mathcal{S}_\kappa((\mathfrak{S}_2)^\kappa_{abc}, (\mathfrak{A}_2)^\kappa_{abc}) = \mathcal{S}((\mathfrak{S}_2)_{abc}, (\mathfrak{A}_2)_{abc})$
and
$\mathcal{S}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc}) = \sup_{0<\kappa\leqslant 1} \mathcal{S}_\kappa((\mathfrak{A}_3)^\kappa_{abc}, (\mathfrak{S}_3)^\kappa_{abc}) =$
$\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{A}_3)^\kappa_{abc1}, (\mathfrak{S}_2)^\kappa_{abc1}|, |(\mathfrak{A}_3)^\kappa_{abc2}, (\mathfrak{S}_3)^\kappa_{abc2}|\}) =$
$\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{S}_3)^\kappa_{abc1}, (\mathfrak{A}_3)^\kappa_{abc1}|, |(\mathfrak{S}_3)^\kappa_{abc2}, (\mathfrak{A}_3)^\kappa_{abc2}|\}) =$
$\sup_{0<\kappa\leqslant 1} \mathcal{S}_\kappa((\mathfrak{S}_3)^\kappa_{abc}, (\mathfrak{A}_3)^\kappa_{abc}) = \mathcal{S}((\mathfrak{S}_3)_{abc}, (\mathfrak{A}_3)_{abc})$
Therefore $\mathcal{S}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc}) =$
$\mathcal{S}((\mathfrak{S}_1)_{abc}, (\mathfrak{A}_1)_{abc}), \mathcal{S}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc}) =$
$\mathcal{S}((\mathfrak{S}_2)_{abc}, (\mathfrak{A}_2)_{abc}), \mathcal{S}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc}) =$
$\mathcal{S}((\mathfrak{S}_1)_{abc}, (\mathfrak{A}_1)_{abc})$ .
Thus, we conclude

$$\bar{d}(\mathfrak{A}, \mathfrak{S})_\Theta = \inf\left\{(\rho, \rho, \rho) \succ (0,0,0) : \sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc})}{\rho}\right)\right] \preccurlyeq (1,1,1) \text{ and } \sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{S}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{S}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc})}{\rho}\right)\right] \preccurlyeq (1,1,1)\right\}$$

$\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{S}_2)^\kappa_{abc1}, (\mathfrak{A}_2)^\kappa_{abc1}|, |(\mathfrak{S}_2)^\kappa_{abc2}, (\mathfrak{A}_2)^\kappa_{abc2}|\}) =$
$\sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa((\mathfrak{S}_2)^\kappa_{abc}, (\mathfrak{A}_2)^\kappa_{abc}) = \mathcal{T}((\mathfrak{S}_2)_{abc}, (\mathfrak{A}_2)_{abc})$
and
$\mathcal{T}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc}) = \sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa((\mathfrak{A}_3)^\kappa_{abc}, (\mathfrak{S}_3)^\kappa_{abc}) =$
$\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{A}_3)^\kappa_{abc1}, (\mathfrak{S}_2)^\kappa_{abc1}|, |(\mathfrak{A}_3)^\kappa_{abc2}, (\mathfrak{S}_3)^\kappa_{abc2}|\}) =$
$\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{S}_3)^\kappa_{abc1}, (\mathfrak{A}_3)^\kappa_{abc1}|, |(\mathfrak{S}_3)^\kappa_{abc2}, (\mathfrak{A}_3)^\kappa_{abc2}|\}) =$
$\sup_{0<\kappa\leqslant 1} \mathcal{T}_\kappa((\mathfrak{S}_3)^\kappa_{abc}, (\mathfrak{A}_3)^\kappa_{abc}) = \mathcal{T}((\mathfrak{S}_3)_{abc}, (\mathfrak{A}_3)_{abc})$.
Therefore $\mathcal{T}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc}) =$
$\mathcal{T}((\mathfrak{S}_1)_{abc}, (\mathfrak{A}_1)_{abc}), \mathcal{T}((\mathfrak{A}_2)_{abc}, (\mathfrak{S}_2)_{abc}) =$
$\mathcal{T}((\mathfrak{S}_2)_{abc}, (\mathfrak{A}_2)_{abc}), \mathcal{T}((\mathfrak{A}_3)_{abc}, (\mathfrak{S}_3)_{abc}) =$
$\mathcal{T}((\mathfrak{S}_1)_{abc}, (\mathfrak{A}_1)_{abc})$ .
By proceeding in the same manner, we obtain,
$\mathcal{S}((\mathfrak{A}_1)_{abc}, (\mathfrak{S}_1)_{abc}) = \sup_{0<\kappa\leqslant 1} \mathcal{S}_\kappa((\mathfrak{A}_1)^\kappa_{abc}, (\mathfrak{S}_1)^\kappa_{abc}) =$
$\sup_{0<\kappa\leqslant 1} (\min\{|(\mathfrak{A}_1)^\kappa_{abc1}, (\mathfrak{S}_1)^\kappa_{abc1}|, |(\mathfrak{A}_1)^\kappa_{abc2}, (\mathfrak{S}_1)^\kappa_{abc2}|\}) =$

$\sup_{0<\ltimes\leqslant 1}(\min\{|(\mathfrak{S}_1)^{\ltimes}_{abc1},(\mathfrak{A}_1)^{\ltimes}_{abc1}|,|(\mathfrak{S}_1)^{\ltimes}_{abc2},(\mathfrak{A}_1)^{\ltimes}_{abc2}|\}) =$
$\sup_{0<\ltimes\leqslant 1}\mathcal{S}_{\ltimes}((\mathfrak{S}_1)^{\ltimes}_{abc},(\mathfrak{A}_1)^{\ltimes}_{abc}) = \mathcal{S}((\mathfrak{S}_1)_{abc},(\mathfrak{A}_1)_{abc})$
and
$\mathcal{S}((\mathfrak{A}_2)_{abc},(\mathfrak{S}_2)_{abc}) = \sup_{0<\ltimes\leqslant 1}\mathcal{S}_{\ltimes}((\mathfrak{A}_2)^{\ltimes}_{abc},(\mathfrak{S}_2)^{\ltimes}_{abc}) =$
$\sup_{0<\ltimes\leqslant 1}(\min\{|(\mathfrak{A}_2)^{\ltimes}_{abc1},(\mathfrak{S}_2)^{\ltimes}_{abc1}|,|(\mathfrak{A}_2)^{\ltimes}_{abc2},(\mathfrak{S}_2)^{\ltimes}_{abc2}|\}) =$
$\sup_{0<\ltimes\leqslant 1}(\min\{|(\mathfrak{S}_2)^{\ltimes}_{abc1},(\mathfrak{A}_2)^{\ltimes}_{abc1}|,|(\mathfrak{S}_2)^{\ltimes}_{abc2},(\mathfrak{A}_2)^{\ltimes}_{abc2}|\}) =$
$\sup_{0<\ltimes\leqslant 1}\mathcal{S}_{\ltimes}((\mathfrak{S}_2)^{\ltimes}_{abc},(\mathfrak{A}_2)^{\ltimes}_{abc}) = \mathcal{S}((\mathfrak{S}_2)_{abc},(\mathfrak{A}_2)_{abc})$
and
$\mathcal{S}((\mathfrak{A}_3)_{abc},(\mathfrak{S}_3)_{abc}) = \sup_{0<\ltimes\leqslant 1}\mathcal{S}_{\ltimes}((\mathfrak{A}_3)^{\ltimes}_{abc},(\mathfrak{S}_3)^{\ltimes}_{abc}) =$
$\sup_{0<\ltimes\leqslant 1}(\min\{|(\mathfrak{A}_3)^{\ltimes}_{abc1},(\mathfrak{S}_2)^{\ltimes}_{abc1}|,|(\mathfrak{A}_3)^{\ltimes}_{abc2},(\mathfrak{S}_3)^{\ltimes}_{abc2}|\}) =$
$\sup_{0<\ltimes\leqslant 1}(\min\{|(\mathfrak{S}_3)^{\ltimes}_{abc1},(\mathfrak{A}_3)^{\ltimes}_{abc1}|,|(\mathfrak{S}_3)^{\ltimes}_{abc2},(\mathfrak{A}_3)^{\ltimes}_{abc2}|\}) =$
$\sup_{0<\ltimes\leqslant 1}\mathcal{S}_{\ltimes}((\mathfrak{S}_3)^{\ltimes}_{abc},(\mathfrak{A}_3)^{\ltimes}_{abc}) = \mathcal{S}((\mathfrak{S}_3)_{abc},(\mathfrak{A}_3)_{abc})$.
Therefore $\mathcal{S}((\mathfrak{A}_1)_{abc},(\mathfrak{S}_1)_{abc}) =$
$\mathcal{S}((\mathfrak{S}_1)_{abc},(\mathfrak{A}_1)_{abc}),\mathcal{S}((\mathfrak{A}_2)_{abc},(\mathfrak{S}_2)_{abc}) =$
$\mathcal{S}((\mathfrak{S}_2)_{abc},(\mathfrak{A}_2)_{abc}),\mathcal{S}((\mathfrak{A}_3)_{abc},(\mathfrak{S}_3)_{abc}) =$
$\mathcal{S}((\mathfrak{S}_1)_{abc},(\mathfrak{A}_1)_{abc})$ .
Thus, we obtain that,
$\bar{d}(\mathfrak{A},\mathfrak{S})_{\Theta} = \inf\{(\rho,\rho,\rho) \succ (0,0,0) :$
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{A}_1)_{abc},(\mathfrak{S}_1)_{abc})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{A}_2)_{abc},(\mathfrak{S}_2)_{abc})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{A}_3)_{abc},(\mathfrak{S}_3)_{abc})}{\rho}\right)\right] \preccurlyeq$
$(1,1,1)$ and $\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{A}_1)_{abc},(\mathfrak{S}_1)_{abc})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{S}((\mathfrak{A}_2)_{abc},(\mathfrak{S}_2)_{abc})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{S}((\mathfrak{A}_3)_{abc},(\mathfrak{S}_3)_{abc})}{\rho}\right)\right] \preccurlyeq$
$(1,1,1)\}$ =
$\inf\{(\rho,\rho,\rho) \succ (0,0,0) : \sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{S}_1)_{abc},(\mathfrak{A}_1)_{abc})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{S}_2)_{abc},(\mathfrak{A}_2)_{abc})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{S}_3)_{abc},(\mathfrak{A}_3)_{abc})}{\rho}\right)\right] \preccurlyeq$
$(1,1,1)$ and $\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{S}_1)_{abc},(\mathfrak{A}_1)_{abc})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{S}((\mathfrak{S}_2)_{abc},(\mathfrak{A}_2)_{abc})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{S}((\mathfrak{S}_3)_{abc},(\mathfrak{A}_3)_{abc})}{\rho}\right)\right] \preccurlyeq$
$(1,1,1)\} = \bar{d}(\mathfrak{S},\mathfrak{A})_{\Theta}$ .
Therefore $\bar{d}(\mathfrak{A},\mathfrak{S})_{\Theta} = \bar{d}(\mathfrak{S},\mathfrak{A})_{\Theta}$ .
iiii) assume that $\rho_1,\rho_2 \succ 0$, then
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{A}_1)_{abc},(\mathfrak{R}_1)_{abc})}{\rho_1}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{A}_2)_{abc},(\mathfrak{R}_2)_{abc})}{\rho_1}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{A}_2)_{abc},(\mathfrak{R}_2)_{abc})}{\rho_1}\right)\right] \preccurlyeq (1,1,1)$,
and
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{R}_1)_{abc},(\mathfrak{S}_1)_{abc})}{\rho_2}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{R}_2)_{abc},(\mathfrak{S}_2)_{abc})}{\rho_2}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{R}_3)_{abc},(\mathfrak{S}_3)_{abc})}{\rho_2}\right)\right] \preccurlyeq (1,1,1)$.
Suppose that $\rho = \rho_1 + \rho_2$ , using $\mathcal{T}'$s definition, we obtain
$\mathcal{T}((\mathfrak{A}_1)_{abc},(\mathfrak{S}_1)_{abc}) =$
$\sup_{0<\ltimes\leqslant 1}\mathcal{T}_{\ltimes}((\mathfrak{A}_1)^{\ltimes}_{abc},(\mathfrak{S}_1)^{\ltimes}_{abc}),\mathcal{T}((\mathfrak{A}_2)_{abc},(\mathfrak{S}_2)_{abc}) =$
$\sup_{0<\ltimes\leqslant 1}\mathcal{T}_{\ltimes}((\mathfrak{A}_2)^{\ltimes}_{abc},(\mathfrak{S}_2)^{\ltimes}_{abc}),\mathcal{T}((\mathfrak{A}_3)_{abc},(3)_{abc}) =$
$\sup_{0<\ltimes\leqslant 1}\mathcal{T}_{\ltimes}((\mathfrak{A}_3)^{\ltimes}_{abc},(\mathfrak{S}_3)^{\ltimes}_{abc})$
and
$\mathcal{T}_{\ltimes}((\mathfrak{A}_1)^{\ltimes}_{abc},(\mathfrak{S}_1)^{\ltimes}_{abc}) = \min\{|(\mathfrak{A}_1)^{\ltimes}_{abc1} - (\mathfrak{S}_1)^{\ltimes}_{abc1}|,|(\mathfrak{A}_1)^{\ltimes}_{abc2} - (\mathfrak{S}_1)^{\ltimes}_{abc2}|\},\mathcal{T}_{\ltimes}((\mathfrak{A}_2)^{\ltimes}_{abc},(\mathfrak{S}_2)^{\ltimes}_{abc}) = \min\{|(\mathfrak{A}_2)^{\ltimes}_{abc1} - (\mathfrak{S}_2)^{\ltimes}_{abc1}|,|(\mathfrak{A}_2)^{\ltimes}_{abc2} - (\mathfrak{S}_2)^{\ltimes}_{abc2}|\},\mathcal{T}_{\ltimes}((\mathfrak{A}_3)^{\ltimes}_{abc},(\mathfrak{S}_3)^{\ltimes}_{abc}) = \min\{|(\mathfrak{A}_3)^{\ltimes}_{abc1} - (\mathfrak{S}_3)^{\ltimes}_{abc1}|,|(\mathfrak{A}_3)^{\ltimes}_{abc2} - (\mathfrak{S}_3)^{\ltimes}_{abc2}|\}$ .
Using $\mathcal{T}$'s definition, we have

$\mathcal{T}_{\ltimes}((\mathfrak{A}_1)^{\ltimes},(\mathfrak{S}_1)^{\ltimes}) \preccurlyeq \mathcal{T}_{\ltimes}((\mathfrak{A}_1)^{\ltimes},(\mathfrak{R}_1)^{\ltimes}) + \mathcal{T}_{\ltimes}((\mathfrak{R}_1)^{\ltimes},(\mathfrak{S}_1)^{\ltimes}),\mathcal{T}_{\ltimes}((\mathfrak{A}_2)^{\ltimes},(\mathfrak{S}_2)^{\ltimes}) \preccurlyeq \mathcal{T}_{\ltimes}((\mathfrak{A}_2)^{\ltimes},(\mathfrak{R}_2)^{\ltimes}) + \mathcal{T}_{\ltimes}((\mathfrak{R}_2)^{\ltimes},(\mathfrak{S}_2)^{\ltimes}),\mathcal{T}_{\ltimes}((\mathfrak{A}_3)^{\ltimes},(\mathfrak{S}_3)^{\ltimes}) \preccurlyeq \mathcal{T}_{\ltimes}((\mathfrak{A}_3)^{\ltimes},(\mathfrak{R}_3)^{\ltimes}) + \mathcal{T}_{\ltimes}((\mathfrak{R}_3)^{\ltimes},(\mathfrak{S}_3)^{\ltimes}), \forall \ltimes \in (0,1]$ .
When we take the supremum of $\ltimes$, we obtain
$\sup_{0<\ltimes\leqslant 1}\mathcal{T}_{\ltimes}((\mathfrak{A}_1)^{\ltimes},(\mathfrak{S}_1)^{\ltimes}) \preccurlyeq \sup_{0\leqslant\ltimes\leqslant 1}\mathcal{T}_{\ltimes}((\mathfrak{A}_1)^{\ltimes},(\mathfrak{R}_1)^{\ltimes}) + \sup_{0\leqslant\ltimes\leqslant 1}\mathcal{T}_{\ltimes}((\mathfrak{R}_1)^{\ltimes},(\mathfrak{S}_1)^{\ltimes}), \sup_{0<\ltimes\leqslant 1}\mathcal{T}_{\ltimes}((\mathfrak{A}_2)^{\ltimes},(\mathfrak{S}_2)^{\ltimes}) \preccurlyeq \sup_{0\leqslant\ltimes\leqslant 1}\mathcal{T}_{\ltimes}((\mathfrak{A}_2)^{\ltimes},(\mathfrak{R}_2)^{\ltimes}) + \sup_{0\leqslant\ltimes\leqslant 1}\mathcal{T}_{\ltimes}((\mathfrak{R}_2)^{\ltimes},(\mathfrak{S}_2)^{\ltimes}), \sup_{0<\ltimes\leqslant 1}\mathcal{T}_{\ltimes}((\mathfrak{A}_3)^{\ltimes},(\mathfrak{S}_3)^{\ltimes}) \preccurlyeq \sup_{0\leqslant\ltimes\leqslant 1}\mathcal{T}_{\ltimes}((\mathfrak{A}_3)^{\ltimes},(\mathfrak{R}_3)^{\ltimes}) + \sup_{0\leqslant\ltimes\leqslant 1}\mathcal{T}_{\ltimes}((\mathfrak{R}_3)^{\ltimes},(\mathfrak{S}_3)^{\ltimes})$.
Moreover,
$\mathcal{T}((\mathfrak{A}_1)_{abc},(\mathfrak{S}_1)_{abc}) \preccurlyeq \mathcal{T}((\mathfrak{A}_1)_{abc},(\mathfrak{R}_1)_{abc}) + \mathcal{T}((\mathfrak{R}_1)_{abc},(\mathfrak{S}_1)_{abc}),\mathcal{T}((\mathfrak{A}_2)_{abc},(\mathfrak{S}_2)_{abc}) \preccurlyeq \mathcal{T}((\mathfrak{A}_2)_{abc},(\mathfrak{R}_2)_{abc}) + \mathcal{T}((\mathfrak{R}_2)_{abc},(\mathfrak{S}_2)_{abc}),\mathcal{T}((\mathfrak{A}_3)_{abc},(\mathfrak{S}_3)_{abc}) \preccurlyeq \mathcal{T}((\mathfrak{A}_3)_{abc},(\mathfrak{R}_3)_{abc}) + \mathcal{T}((\mathfrak{R}_3)_{abc},(\mathfrak{S}_3)_{abc})$ .
Based on $\Theta'$s continuity, we determine that,
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{A}_1)_{abc},(\mathfrak{S}_1)_{abc})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{A}_2)_{abc},(\mathfrak{S}_2)_{abc})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{A}_3)_{abc},(\mathfrak{S}_3)_{abc})}{\rho}\right)\right]$
$\preccurlyeq \sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{A}_1)_{abc},(\mathfrak{R}_1)_{abc})}{\rho_1+\rho_2} + \frac{\mathcal{T}((\mathfrak{R}_1)_{abc},(\mathfrak{S}_1)_{abc})}{\rho_1+\rho_2}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{A}_2)_{abc},(\mathfrak{R}_2)_{abc})}{\rho_1+\rho_2} + \frac{\mathcal{T}((\mathfrak{R}_2)_{abc},(\mathfrak{S}_2)_{abc})}{\rho_1+\rho_2}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{A}_3)_{abc},(\mathfrak{R}_3)_{abc})}{\rho_1+\rho_2} + \frac{\mathcal{T}((\mathfrak{R}_3)_{abc},(\mathfrak{S}_3)_{abc})}{\rho_1+\rho_2}\right)\right]$
$\preccurlyeq \sup_{abc}\left[\Theta_1\left[\left(\frac{\rho_1}{\rho_1+\rho_2}\right)\left(\frac{\mathcal{T}((\mathfrak{A}_1)_{abc},(\mathfrak{R}_1)_{abc})}{\rho_1}\right) + \left(\frac{\rho_2}{\rho_1+\rho_2}\right)\left(\frac{\mathcal{T}((\mathfrak{R}_1)_{abc},(\mathfrak{S}_1)_{abc})}{\rho_2}\right)\right] \curlyvee \Theta_2\left[\left(\frac{\rho_1}{\rho_1+\rho_2}\right)\left(\frac{\mathcal{T}((\mathfrak{A}_2)_{abc},(\mathfrak{R}_2)_{abc})}{\rho_1}\right) + \left(\frac{\rho_2}{\rho_1+\rho_2}\right)\left(\frac{\mathcal{T}((\mathfrak{R}_2)_{abc},(\mathfrak{S}_2)_{abc})}{\rho_2}\right)\right] \curlyvee \Theta_3\left[\left(\frac{\rho_1}{\rho_1+\rho_2}\right)\left(\frac{\mathcal{T}((\mathfrak{A}_3)_{abc},(\mathfrak{R}_3)_{abc})}{\rho_1}\right) + \left(\frac{\rho_2}{\rho_1+\rho_2}\right)\left(\frac{\mathcal{T}((\mathfrak{R}_3)_{abc},(\mathfrak{S}_3)_{abc})}{\rho_2}\right)\right]\right]$

$\preccurlyeq \sup_{abc}\left(\frac{\rho_1}{\rho_1+\rho_2}\right)\left[\Theta_1\left[\left(\frac{\mathcal{T}((\mathfrak{A}_1)_{abc},(\Re_1)_{abc})}{\rho_1}\right)\right]\curlyvee \Theta_2\left[\left(\frac{\mathcal{T}((\mathfrak{A}_2)_{abc},(\Re_2)_{abc})}{\rho_1}\right)\right]\curlyvee \Theta_3\left[\left(\frac{\mathcal{T}((\mathfrak{A}_3)_{abc},(\Re_3)_{abc})}{\rho_1}\right)\right]\right]$

$+ \sup_{abc}\left(\frac{\rho_2}{\rho_1+\rho_2}\right)\left[\Theta_1\left[\left(\frac{\mathcal{T}((\Re_1)_{abc},(\Im_1)_{abc})}{\rho_2}\right)\right]\curlyvee \Theta_2\left[\left(\frac{\mathcal{T}((\Re_2)_{abc},(\Im_2)_{abc})}{\rho_2}\right)\right]\curlyvee \Theta_3\left[\left(\frac{\mathcal{T}((\Re_3)_{abc},(\Im_3)_{abc})}{\rho_2}\right)\right]\right] \preccurlyeq (1,1,1).$

Given that $\rho's$ are non-negative, then the infimum of these $\rho's$ is introduced by

$\inf\left\{(\rho,\rho,\rho)\succ(0,0,0): \sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{A}_1)_{abc},(\Im_1)_{abc})}{\rho}\right)\curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{A}_2)_{abc},(\Im_2)_{abc})}{\rho}\right)\curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{A}_3)_{abc},(\Im_3)_{abc})}{\rho}\right)\right]\preccurlyeq (1,1,1)\right\}$

$\preccurlyeq \inf\left\{(\rho_1,\rho_1,\rho_1)\succ(0,0,0): \sup_{abc}\left[\Theta_1\left[\left(\frac{\mathcal{T}((\mathfrak{A}_1)_{abc},(\Re_1)_{abc})}{\rho_1}\right)\right]\curlyvee \Theta_2\left[\left(\frac{\mathcal{T}((\mathfrak{A}_2)_{abc},(\Re_2)_{abc})}{\rho_1}\right)\right]\curlyvee \Theta_3\left[\left(\frac{\mathcal{T}((\mathfrak{A}_3)_{abc},(\Re_3)_{abc})}{\rho_1}\right)\right]\preccurlyeq (1,1,1)\right\}$

$+\inf\left\{(\rho_2,\rho_2,\rho_2)\succ(0,0,0): \sup_{abc}\left[\Theta_1\left[\left(\frac{\mathcal{T}((\Re_1)_{abc},(\Im_1)_{abc})}{\rho_2}\right)\right]\curlyvee \Theta_2\left[\left(\frac{\mathcal{T}((\Re_2)_{abc},(\Im_2)_{abc})}{\rho_2}\right)\right]\curlyvee \Theta_3\left[\left(\frac{\mathcal{T}((\Re_3)_{abc},(\Im_3)_{abc})}{\rho_2}\right)\right]\preccurlyeq (1,1,1)\right\}.$

Following the same path, we eventually arrive at

$\inf\left\{(\rho,\rho,\rho)\succ(0,0,0): \sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{A}_1)_{abc},(\Im_1)_{abc})}{\rho}\right)\curlyvee \Theta_2\left(\frac{\mathcal{S}((\mathfrak{A}_2)_{abc},(\Im_2)_{abc})}{\rho}\right)\curlyvee \Theta_3\left(\frac{\mathcal{S}((\mathfrak{A}_3)_{abc},(\Im_3)_{abc})}{\rho}\right)\right]\preccurlyeq (1,1,1)\right\}$

$\preccurlyeq \inf\left\{(\rho_1,\rho_1,\rho_1)\succ(0,0,0): \sup_{abc}\left[\Theta_1\left[\left(\frac{\mathcal{S}((\mathfrak{A}_1)_{abc},(\Re_1)_{abc})}{\rho_1}\right)\right]\curlyvee \Theta_2\left[\left(\frac{\mathcal{S}((\mathfrak{A}_2)_{abc},(\Re_2)_{abc})}{\rho_1}\right)\right]\curlyvee \Theta_3\left[\left(\frac{\mathcal{S}((\mathfrak{A}_3)_{abc},(\Re_3)_{abc})}{\rho_1}\right)\right]\preccurlyeq (1,1,1)\right\}$

$+\inf\left\{(\rho_2,\rho_2,\rho_2)\succ(0,0,0): \sup_{abc}\left[\Theta_1\left[\left(\frac{\mathcal{S}((\Re_1)_{abc},(\Im_1)_{abc})}{\rho_2}\right)\right]\curlyvee \Theta_2\left[\left(\frac{\mathcal{S}((\Re_2)_{abc},(\Im_2)_{abc})}{\rho_2}\right)\right]\curlyvee \Theta_3\left[\left(\frac{\mathcal{S}((\Re_3)_{abc},(\Im_3)_{abc})}{\rho_2}\right)\right]\preccurlyeq (1,1,1)\right\}.$

Then,

$\inf\left\{(\rho,\rho,\rho)\succ(0,0,0): \sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{A}_1)_{abc},(\Im_1)_{abc})}{\rho}\right)\curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{A}_2)_{abc},(\Im_2)_{abc})}{\rho}\right)\curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{A}_3)_{abc},(\Im_3)_{abc})}{\rho}\right)\right]\preccurlyeq (1,1,1)\right.$ and $\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{A}_1)_{abc},(\Im_1)_{abc})}{\rho}\right)\curlyvee \Theta_2\left(\frac{\mathcal{S}((\mathfrak{A}_2)_{abc},(\Im_2)_{abc})}{\rho}\right)\curlyvee \Theta_3\left(\frac{\mathcal{S}((\mathfrak{A}_3)_{abc},(\Im_3)_{abc})}{\rho}\right)\right]\preccurlyeq (1,1,1)\Big\}$

$\preccurlyeq$

$\inf\left\{(\rho_1,\rho_1,\rho_1)\succ(0,0,0): \sup_{abc}\left[\Theta_1\left[\left(\frac{\mathcal{T}((\mathfrak{A}_1)_{abc},(\Re_1)_{abc})}{\rho_1}\right)\right]\curlyvee \Theta_2\left[\left(\frac{\mathcal{T}((\mathfrak{A}_2)_{abc},(\Re_2)_{abc})}{\rho_1}\right)\right]\curlyvee \Theta_3\left[\left(\frac{\mathcal{T}((\mathfrak{A}_3)_{abc},(\Re_3)_{abc})}{\rho_1}\right)\right]\preccurlyeq (1,1,1)\right.$, and $\sup_{abc}\left[\Theta_1\left[\left(\frac{\mathcal{S}((\mathfrak{A}_1)_{abc},(\Re_1)_{abc})}{\rho_1}\right)\right]\curlyvee \Theta_2\left[\left(\frac{\mathcal{S}((\mathfrak{A}_2)_{abc},(\Re_2)_{abc})}{\rho_1}\right)\right]\curlyvee \Theta_3\left[\left(\frac{\mathcal{S}((\mathfrak{A}_3)_{abc},(\Re_3)_{abc})}{\rho_1}\right)\right]\preccurlyeq (1,1,1)\Big\}+\inf\left\{(\rho_2,\rho_2,\rho_2)\succ(0,0,0): \sup_{abc}\left[\Theta_1\left[\left(\frac{\mathcal{T}((\Re_1)_{abc},(\Im_1)_{abc})}{\rho_2}\right)\right]\curlyvee \Theta_2\left[\left(\frac{\mathcal{T}((\Re_2)_{abc},(\Im_2)_{abc})}{\rho_2}\right)\right]\curlyvee \Theta_3\left[\left(\frac{\mathcal{T}((\Re_3)_{abc},(\Im_3)_{abc})}{\rho_2}\right)\right]\preccurlyeq (1,1,1)\right.$ and $\sup_{abc}\left[\Theta_1\left[\left(\frac{\mathcal{T}((\Re_1)_{abc},(\Im_1)_{abc})}{\rho_2}\right)\right]\curlyvee \Theta_2\left[\left(\frac{\mathcal{T}((\Re_2)_{abc},(\Im_2)_{abc})}{\rho_2}\right)\right]\curlyvee \Theta_3\left[\left(\frac{\mathcal{T}((\Re_3)_{abc},(\Im_3)_{abc})}{\rho_2}\right)\right]\preccurlyeq (1,1,1)\Big\}.$

Therefore $\bar{d}(\mathfrak{A},\Im)_\Theta \preccurlyeq \bar{d}(\mathfrak{A},\Re)_\Theta + \bar{d}(\Re,\Im)_\Theta$.

Thus,

$(\ell_\infty)^3_{\mathbb{F}}(\Theta)$ is metric space.

**Theorem 3.2:**

Let $(\ell_\infty)^3_{\mathbb{F}}(\Theta)$ be a complete space under the metric:

$\bar{d}(\mathfrak{A},\Im)_\Theta = \inf\left\{(\rho,\rho,\rho)\succ(0,0,0): \sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{A}_1)_{abc},(\Im_1)_{abc})}{\rho}\right)\curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{A}_2)_{abc},(\Im_2)_{abc})}{\rho}\right)\curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{A}_3)_{abc},(\Im_3)_{abc})}{\rho}\right)\right]\preccurlyeq (1,1,1)\right.$, and $\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{A}_1)_{abc},(\Im_1)_{abc})}{\rho}\right)\curlyvee \Theta_2\left(\frac{\mathcal{S}((\mathfrak{A}_2)_{abc},(\Im_2)_{abc})}{\rho}\right)\curlyvee \Theta_3\left(\frac{\mathcal{S}((\mathfrak{A}_3)_{abc},(\Im_3)_{abc})}{\rho}\right)\right]\preccurlyeq (1,1,1)\Big\}, \forall\ \mathfrak{A}=(\mathfrak{A}_1,\mathfrak{A}_2,\mathfrak{A}_3), \Im=(\Im_1,\Im_2,\Im_3)\in (\ell_\infty)^3_{\mathbb{F}}(\Theta).$

**Proof:**

Assume that $\left((\Re_1)^{(jih)}\right),\left((\Re_2)^{(jih)}\right)$ and $\left((\Re_3)^{(jih)}\right)$ are Cauchy triple sequence in $(\ell_\infty)^3_{\mathbb{F}}(\Theta) \ni (\Re_1)^{(jih)} =$

$\left((\Re_1)_{uts}^{(jih)}\right)_{u,t,s=1}^{\infty}$ and $(\Re_1)^{(jih)} =$
$\left((\Re_1)_{uts}^{(jih)}\right)_{u,t,s=1}^{\infty}$ and $(\Re_1)^{(jih)} = \left((\Re_1)_{uts}^{(jih)}\right)_{u,t,s=1}^{\infty}$ .
Let $\varepsilon \succ 0$. For a fixed exist $x_0 \succ 0$, choose $\mathfrak{p} \succ 0 \ni$ $\left[\Theta_1\left(\frac{\mathfrak{p}x_0}{2}\right) \curlyvee \Theta_2\left(\frac{\mathfrak{p}x_0}{2}\right) \curlyvee \Theta_3\left(\frac{\mathfrak{p}x_0}{2}\right)\right] \succcurlyeq (1,1,1)$ . $\exists$ a positive integer $n_0 = n_0(\varepsilon) \ni$
$\bar{d}\begin{pmatrix}((\Re_1)^{(jih)}, (\Re_1)^{(fed)}), ((\Re_2)^{(jih)}, (\Re_2)^{(fed)}),\\ ((\Re_3)^{(jih)}, (\Re_3)^{(fed)})\end{pmatrix}_{\mathbb{M}} \prec$
$\left(\frac{\varepsilon}{\mathfrak{p}x_0}, \frac{\varepsilon}{\mathfrak{p}x_0}, \frac{\varepsilon}{\mathfrak{p}x_0}\right), \forall\, j,i,\hbar,f,e,d \succcurlyeq n_0.$
Using $\bar{d}_\Theta$'s definition, we obtain
$\inf\left\{(\rho,\rho,\rho) \succ (0,0,0) : \right.$
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}\left((\Re_1)_{abc}^{(jih)},\ (\Re_1)_{abc}^{(fed)}\right)}{\rho}\right) \curlyvee \right.$
$\Theta_2\left(\frac{\mathcal{T}\left((\Re_2)_{abc}^{(jih)},\ (\Re_2)_{abc}^{(fed)}\right)}{\rho}\right) \curlyvee$
$\left.\left.\Theta_3\left(\frac{\mathcal{T}\left((\Re_3)_{abc}^{(jih)},\ (\Re_3)_{abc}^{(fed)}\right)}{\rho}\right)\right] \preccurlyeq$
$(1,1,1)$ and $\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}\left((\Re_1)_{abc}^{(jih)},\ (\Re_1)_{abc}^{(fed)}\right)}{\rho}\right) \curlyvee \right.$
$\Theta_2\left(\frac{\mathcal{S}\left((\Re_2)_{abc}^{(jih)},\ (\Re_2)_{abc}^{(fed)}\right)}{\rho}\right) \curlyvee$
$\left.\left.\Theta_3\left(\frac{\mathcal{S}\left((\Re_3)_{abc}^{(jih)},\ (\Re_3)_{abc}^{(fed)}\right)}{\rho}\right)\right] \preccurlyeq (1,1,1)\right\} \prec$
$(\varepsilon,\varepsilon,\varepsilon), \forall\, j,i,\hbar,f,e,d \succcurlyeq n_0 \ldots\ldots\ldots (1),$
which leads to
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}\left((\Re_1)_{abc}^{(jih)},\ (\Re_1)_{abc}^{(fed)}\right)}{\rho}\right) \curlyvee \right.$
$\Theta_2\left(\frac{\mathcal{T}\left((\Re_2)_{abc}^{(jih)},\ (\Re_2)_{abc}^{(fed)}\right)}{\rho}\right) \curlyvee$
$\left.\Theta_3\left(\frac{\mathcal{T}\left((\Re_3)_{abc}^{(jih)},\ (\Re_3)_{abc}^{(fed)}\right)}{\rho}\right)\right] \preccurlyeq (1,1,1) \ldots\ldots (2)$ .
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}\left((\Re_1)_{abc}^{(jih)},\ (\Re_1)_{abc}^{(fed)}\right)}{\rho}\right) \curlyvee \right.$
$\Theta_2\left(\frac{\mathcal{S}\left((\Re_2)_{abc}^{(jih)},\ (\Re_2)_{abc}^{(fed)}\right)}{\rho}\right) \curlyvee$
$\left.\Theta_3\left(\frac{\mathcal{S}\left((\Re_3)_{abc}^{(jih)},\ (\Re_3)_{abc}^{(fed)}\right)}{\rho}\right)\right] \preccurlyeq (1,1,1) \ldots\ldots\ldots (3)$ .
From (2), we have
$\left[\Theta_1\left(\frac{\mathcal{T}\left((\Re_1)_{abc}^{(jih)},\ (\Re_1)_{abc}^{(fed)}\right)}{\bar{d}\left((\Re_1)^{(jih)},(\Re_1)^{(fed)}\right)}\right) \curlyvee \right.$
$\Theta_2\left(\frac{\mathcal{T}\left((\Re_2)_{abc}^{(jih)},\ (\Re_2)_{abc}^{(fed)}\right)}{\bar{d}\left((\Re_2)^{(jih)},(\Re_2)^{(fed)}\right)}\right) \curlyvee$
$\left.\Theta_3\left(\frac{\mathcal{T}\left((\Re_3)_{abc}^{(jih)},\ (\Re_3)_{abc}^{(fed)}\right)}{\bar{d}\left((\Re_3)^{(jih)},(\Re_3)^{(fed)}\right)}\right)\right] \preccurlyeq (1,1,1) \preccurlyeq \left[\Theta_1\left(\frac{\mathfrak{p}x_0}{2}\right) \curlyvee \right.$
$\left.\Theta_2\left(\frac{\mathfrak{p}x_0}{2}\right) \curlyvee \Theta_3\left(\frac{\mathfrak{p}x_0}{2}\right)\right]$ .
By $\Theta'$s continuity, we determine that,

$\mathcal{T}\begin{pmatrix}\left((\Re_1)_{abc}^{(jih)}, (\Re_1)_{abc}^{(fed)}\right), \left((\Re_2)_{abc}^{(jih)}, (\Re_2)_{abc}^{(fed)}\right),\\ \left((3)_{abc}^{(jih)}, (\Re_3)_{abc}^{(fed)}\right)\end{pmatrix} \preccurlyeq$
$\left(\frac{\mathfrak{p}x_0}{2}, \frac{\mathfrak{p}x_0}{2}, \frac{\mathfrak{p}x_0}{2}\right) \cdot \left(\frac{\varepsilon}{\mathfrak{p}x_0}, \frac{\varepsilon}{\mathfrak{p}x_0}, \frac{\varepsilon}{\mathfrak{p}x_0}\right) = \left(\frac{\varepsilon}{2}, \frac{\varepsilon}{2}, \frac{\varepsilon}{2}\right)$ .
According to the completeness property of $\mathbb{R}(\mathbb{I})$, $\left((\Re_1)_{abc}^{(jih)}\right), \left((\Re_2)_{abc}^{(jih)}\right), \left((\Re_3)_{abc}^{(jih)}\right)$ is convergent in $\mathbb{R}(\mathbb{I})$, since it is a Cauchy triple sequence in $\mathbb{R}(\mathbb{I})$ .
Let $\lim_{jih}(\Re_1)_{abc}^{(jih)} = (\Re_1)_{abc}$ , $\lim_{jih}(\Re_2)_{abc}^{(jih)} =$
$(\Re_2)_{abc}$ , $\lim_{jih}(\Re_3)_{abc}^{(jih)} = (\Re_3)_{abc}$ , $\forall\, a,b,c,e \in \mathbb{N}$.
We must demonstrate that, $\lim_{jih}(\Re_1)^{(jih)} =$
$\Re_1$ , $\lim_{jih}(\Re_2)^{(jih)} = \Re_2$ , $\lim_{jih}(\Re_3)^{(jih)} =$
$\Re_3$ , $\forall\, \Re_1, \Re_2, \Re_3 \in (\ell_\infty)_{\mathbb{F}}^3(\Theta)$
$\Theta$ being a continuous, taking $f, e, d \to$
$\infty$ and fixing $j, i, \hbar$. From (2), we obtain
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}\left((\Re_1)_{abc}^{(jih)}, (\Re_1)_{abc}\right)}{\rho}\right) \curlyvee \right.$
$\left.\Theta_2\left(\frac{\mathcal{T}\left((\Re_2)_{abc}^{(jih)}, (\Re_2)_{abc}\right)}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}\left((\Re_3)_{abc}^{(jih)}, (\Re_3)_{abc}\right)}{\rho}\right)\right] \preccurlyeq$
$(1,1,1)$, for some $\rho \succ 0, \forall\, j,i,\hbar \succcurlyeq n_0$.
Continuing in the same manner, from (3), we have
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}\left((\Re_1)_{abc}^{(jih)}, (\Re_1)_{abc}\right)}{\rho}\right) \curlyvee \right.$
$\left.\Theta_2\left(\frac{\mathcal{S}\left((\Re_2)_{abc}^{(jih)}, (\Re_2)_{abc}\right)}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{S}\left((\Re_3)_{abc}^{(jih)}, (\Re_3)_{abc}\right)}{\rho}\right)\right] \preccurlyeq$
$(1,1,1)$, for some $\rho \succ 0, \forall\, j,i,\hbar \succcurlyeq n_0$.
Now, taking the infimum for $\rho'$s, from (1), we obtain
$\inf\left\{(\rho,\rho,\rho) \succ (0,0,0) : \sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}\left((\Re_1)_{abc}^{(jih)}, (\Re_1)_{abc}\right)}{\rho}\right) \curlyvee \right.\right.$
$\left.\Theta_2\left(\frac{\mathcal{T}\left((\Re_2)_{abc}^{(jih)}, (\Re_2)_{abc}\right)}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}\left((\Re_3)_{abc}^{(jih)}, (\Re_3)_{abc}\right)}{\rho}\right)\right] \preccurlyeq$
$(1,1,1)$, and $\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}\left((\Re_1)_{abc}^{(jih)}, (\Re_1)_{abc}\right)}{\rho}\right) \curlyvee \right.$
$\left.\left.\Theta_2\left(\frac{\mathcal{S}\left((\Re_2)_{abc}^{(jih)}, (\Re_2)_{abc}\right)}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{S}\left((\Re_3)_{abc}^{(jih)}, (\Re_3)_{abc}\right)}{\rho}\right)\right] \preccurlyeq (1,1,1)\right\} \prec (\varepsilon,\varepsilon,\varepsilon), \forall\, j,i,\hbar \succcurlyeq n_0,$
which tends to
$\bar{d}\left(((\Re_1)^{(jih)}, \Re_1), ((\Re_2)^{(jih)}, \Re_2), ((\Re_3)^{(jih)}, \Re_3)\right)_\Theta \prec$
$(\varepsilon,\varepsilon,\varepsilon), \forall\, j,i,\hbar \succcurlyeq n_0 \Longrightarrow \lim_{jih}(\Re_1)^{(jih)} =$
$\Re_1$ , $\lim_{jih}(\Re_2)^{(jih)} = \Re_2$, $\lim_{jih}(\Re_3)^{(jih)} = \Re_3$
Now, It' prove that $\Re_1, \Re_2, \Re_3 \in (\ell_\infty)_{\mathbb{F}}^3(\Theta)$ .
Taking into account that,
$\bar{d}\left((\Re_1, 0), (\Re_2, 0), (\Re_3, 0)\right)_\Theta \preccurlyeq$
$\bar{d}\left((\Re_1, (\Re_1)^{(jih)}), (\Re_2, (\Re_2)^{(jih)}), (\Re_3, (\Re_3)^{(jih)})\right)_\Theta +$

$\bar{d}\left(\left((\Re_1)^{(jih)},0\right),\left((\Re_1)^{(jih)},0\right),\left((\Re_1)^{(jih)},0\right)\right)_\Theta \prec (\varepsilon,\varepsilon,\varepsilon)+(\Theta_1,\Theta_2,\Theta_3), \forall\, j,i,h \geqslant n_0(\varepsilon)$ ,
We conclude that $\bar{d}\left((\Re_1,0),(\Re_2,0),(\Re_3,0)\right)_\Theta$ is finite.
Therefore $\Re_1,\Re_2,\Re_3 \in (\ell_\infty)_{\mathbb{F}}^3(\Theta)$.
Thus ,
$(\ell_\infty)_{\mathbb{F}}^3(\Theta)$ is complete .

**Theorem3.3:**
$(\ell_\infty)_{\mathbb{F}}^3(\Theta)$ is solid.
**Proof:**
Suppose that $(\mathfrak{M}_{abc}) \in (\ell_\infty)_{\mathbb{F}}^3(\Theta)$. Then we have $\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{M}_1)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{M}_2)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{M}_3)_{abc},\bar{0})}{\rho}\right)\right] \prec \infty$ and $\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{M}_1)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{S}((\mathfrak{M}_2)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{S}((\mathfrak{M}_3)_{abc},\bar{0})}{\rho}\right)\right] \prec \infty$, for some $\rho \succ 0$.
Suppose $(\mathfrak{N}_{abc})$ is a sequence of fuzzy numbers with,
$[\mathcal{A}((\mathfrak{N}_1)_{abc},\bar{0})]_\ltimes = [\mathcal{T}_\ltimes((\mathfrak{N}_1)^\ltimes_{abc},0),\mathcal{S}_\ltimes((\mathfrak{N}_1)^\ltimes_{abc},0)]$ and
$[\mathcal{A}((\mathfrak{N}_2)_{abc},\bar{0})]_\ltimes = [\mathcal{T}_\ltimes((\mathfrak{N}_2)^\ltimes_{abc},0),\mathcal{S}_\ltimes((\mathfrak{N}_2)^\ltimes_{abc},0)]$ and
$[\mathcal{A}((\mathfrak{N}_3)_{abc},\bar{0})]_\ltimes = [\mathcal{T}_\ltimes((\mathfrak{N}_1)^\ltimes_{abc},0),\mathcal{S}_\ltimes((\mathfrak{N}_1)^\ltimes_{abc},0)], \forall\, 0 \prec \ltimes \leqslant 1$
Such that ,
$\mathcal{T}[((\mathfrak{N}_1)_{abc},\bar{0})] \leqslant \mathcal{T}[((\mathfrak{M}_1)_{abc},\bar{0})]$ and
$\mathcal{T}[(\mathfrak{N}_2)_{abc},\bar{0}], \leqslant \mathcal{T}[(\mathfrak{M}_2)_{abc},\bar{0}]$ and
$\mathcal{T}[(\mathfrak{N}_3)_{abc},\bar{0}], \leqslant \mathcal{T}[(\mathfrak{M}_3)_{abc},\bar{0}]$
and
$\mathcal{S}[((\mathfrak{N}_1)_{abc},\bar{0})] \leqslant \mathcal{S}[((\mathfrak{M}_1)_{abc},\bar{0})]$ and
$\mathcal{S}[(\mathfrak{N}_2)_{abc},\bar{0}], \leqslant \mathcal{S}[(\mathfrak{M}_2)_{abc},\bar{0}]$ and
$\mathcal{S}[(\mathfrak{N}_3)_{abc},\bar{0}], \leqslant \mathcal{S}[(\mathfrak{M}_3)_{abc},\bar{0}]$.
Since $\Theta$ is a continuous and not diminishing, we get, for some $\rho \succ 0$ ,
$\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{N}_1)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{N}_2)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{N}_3)_{abc},\bar{0})}{\rho}\right)\right] \leqslant \left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{M}_1)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{M}_2)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{M}_3)_{abc},\bar{0})}{\rho}\right)\right]$,
and
$\left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{N}_1)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{S}((\mathfrak{N}_2)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{S}((\mathfrak{N}_3)_{abc},\bar{0})}{\rho}\right)\right] \leqslant \left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{M}_1)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{S}((\mathfrak{M}_2)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{S}((\mathfrak{M}_3)_{abc},\bar{0})}{\rho}\right)\right]$ .
In addition,
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{N}_1)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{N}_2)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{N}_3)_{abc},\bar{0})}{\rho}\right)\right] \leqslant \sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{M}_1)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{M}_2)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{M}_3)_{abc},\bar{0})}{\rho}\right)\right] \prec \infty$ , for some $\rho \succ 0$ .
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{N}_1)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{S}((\mathfrak{N}_2)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{S}((\mathfrak{N}_3)_{abc},\bar{0})}{\rho}\right)\right] \leqslant \sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{M}_1)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{S}((\mathfrak{M}_2)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{S}((\mathfrak{M}_3)_{abc},\bar{0})}{\rho}\right)\right] \prec \infty$ , for some $\rho \succ 0$ .
Moreover , we have
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{N}_1)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{N}_2)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{N}_3)_{abc},\bar{0})}{\rho}\right)\right] \prec \infty$ and $\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{N}_1)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{S}((\mathfrak{N}_2)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{S}((\mathfrak{N}_3)_{abc},\bar{0})}{\rho}\right)\right] \prec \infty$ .
Therefore $(\mathfrak{N}_{abc}) \in (\ell_\infty)_{\mathbb{F}}^3(\Theta)$ .
Thus ,
$(\ell_\infty)_{\mathbb{F}}^3(\Theta)$ is solid .

**Theorem 3.4:**
$(\ell_\infty)_{\mathbb{F}}^3(\Theta)$ is symmetric.
**Proof:**
Assume $(\mathfrak{M}_{abc}) \in (\ell_\infty)_{\mathbb{F}}^3(\Theta)$ and $(\mathfrak{N}_{abc})$ is a reorganized of $(\mathfrak{M}_{abc}) \ni \mathfrak{M}_{abc} = \mathfrak{N}_{qpn_{abc}}, \forall\, \mathfrak{a},\mathfrak{b},\mathfrak{c} \in \mathbb{N}$ .
Then, we have
$\mathcal{T}\left(((\mathfrak{N}_1)_{qpn_{abc}},\bar{0}),((\mathfrak{N}_2)_{qpn_{abc}},\bar{0}),((\mathfrak{N}_3)_{qpn_{abc}},\bar{0})\right) = \mathcal{T}\left(((\mathfrak{M}_1)_{abc},\bar{0}),((\mathfrak{M}_2)_{abc},\bar{0}),((\mathfrak{M}_2)_{abc},\bar{0})\right)$,
and
$\mathcal{S}\left(((\mathfrak{N}_1)_{qpn_{abc}},\bar{0}),((\mathfrak{N}_2)_{qpn_{abc}},\bar{0}),((\mathfrak{N}_3)_{qpn_{abc}},\bar{0})\right) = \mathcal{S}\left(((\mathfrak{M}_1)_{abc},\bar{0}),((\mathfrak{M}_2)_{abc},\bar{0}),((\mathfrak{M}_2)_{abc},\bar{0})\right)$ .
Based on $\Theta'$s continuity, we determine that,
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{N}_1)_{qpn_{abc}},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{N}_2)_{qpn_{abc}},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{N}_3)_{qpn_{abc}},\bar{0})}{\rho}\right)\right] = \sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{M}_1)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{M}_2)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{M}_3)_{abc},\bar{0})}{\rho}\right)\right]$, for some $\rho \succ 0$ ,
and
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{N}_1)_{qpn_{abc}},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{S}((\mathfrak{N}_2)_{qpn_{abc}},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{S}((\mathfrak{N}_3)_{qpn_{abc}},\bar{0})}{\rho}\right)\right] = \sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{M}_1)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{S}((\mathfrak{M}_2)_{abc},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{S}((\mathfrak{M}_3)_{abc},\bar{0})}{\rho}\right)\right]$ , for some $\rho \succ 0$ .
This means that ,
$\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{T}((\mathfrak{N}_1)_{qpn_{abc}},\bar{0})}{\rho}\right) \curlyvee \Theta_2\left(\frac{\mathcal{T}((\mathfrak{N}_2)_{qpn_{abc}},\bar{0})}{\rho}\right) \curlyvee \Theta_3\left(\frac{\mathcal{T}((\mathfrak{N}_3)_{qpn_{abc}},\bar{0})}{\rho}\right)\right] \prec (\infty,\infty,\infty)$ and $\sup_{abc}\left[\Theta_1\left(\frac{\mathcal{S}((\mathfrak{N}_1)_{qpn_{abc}},\bar{0})}{\rho}\right) \curlyvee$

$$\Theta_2\left(\frac{s\left((\mathfrak{R}_2)_{qpn_{abc}},\bar{0}\right)}{\rho}\right) \curlyvee \Theta_3\left(\frac{s\left((\mathfrak{R}_3)_{qpn_{abc}},\bar{0}\right)}{\rho}\right)\Bigg] \prec (\infty,\infty,\infty),$$

for some $\rho \succ 0$.

Therefore $(\mathfrak{R}_{abc}) \in (\ell_\infty)^3_{\mathbb{F}}(\Theta)$.

Thus ,

$(\ell_\infty)^3_{\mathbb{F}}(\Theta)$ is symmetric .

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Arabic Abstract

سوف نقدم دالة اوليسز المطلقة الناقصة البسيطة الثلاثية في هذا البحث , والمحددة بواسطة فضاءات المتتابعات الثلاثية مع المترية الضبابية , وكذلك سوف نناقش بعض الخواص , مثلا الفضاء $(\ell_\infty)^3_{\mathbb{F}}(\Theta)$ هو فضاء متناظر , فضاء صلب , فضاء كامل .


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