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Research Article

New Types of Fuzzy Ideals in KU-algebra

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Abstract

We introduce in this research, new types of fuzzy ideals of an algebra of type KU : Fuzzy Complete Ideal, Fuzzy E-Ideal and Fuzzy Complete E-Ideal, respectively. In addition, some propositions explained relationships with these fuzzy ideals types.

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1. Introduction

In [10] , Leerawat and Prabpayak introduced congruences and ideals in algebras of type BCC [1,6], also studied and introduced a new algebraic structure which is denoted by algebra of type KU or KU-algebra, also investigated some related properties. Jawad and AL-Shaher [7] studied new types of ideals of KU-algebras, and also stated the relationship between them. Zadeh [13] introduced the notion of fuzzy sets. Dudek et al studied fuzzy ideals, also several fuzzy structures in BCC-algebras are considered [2,3,4]. Moreover, Mostafa et al [8] studied the notion of fuzzy KU-ideals of KU-algebras and then they investigated several basic properties which are related to fuzzy KU-ideals. Mostafa and Kareem [9] introduced fuzzy n-fold KU-ideals of KU-algebras and obtained some related results. The aim of this paper is to introduce fuzzy complete ideal, fuzzy
E-ideal, fuzzy complete E-ideal in KU-algebras, also some of the basic properties are investigated.

2. Basic Concepts and Notations

In this section, we introduce definitions in KU-g , B.D KU-involutory , complete ideal , E-ideal , complete E-ideal , fuzzy KU-ideal, fuzzy KU-subalgebra, fuzzy ideal, and some of their properties.

Definition $(\mathbf{2.1})[\mathbf{10,11}]$

Algebra $(Y,\diamond,0)$ with type $(2,0)$ is called KU-algebra and we will denote by KU-g , if it hold the axioms:

(1) $(\mathfrak{r}\diamond\mathfrak{s})\diamond\big((\mathfrak{s}\diamond\mathfrak{t})\diamond(\mathfrak{r}\diamond\mathfrak{t})\big)=constant\ 0$, $\forall\,\mathfrak{r},\mathfrak{s},\mathfrak{t}\in Y$

(2) $\mathfrak{r}\diamond 0=constant\ 0$

(3) $constant\ 0\diamond\mathfrak{r}=\mathfrak{r}$

(4) $\mathfrak{r}\diamond\mathfrak{s}=constant\ 0$ and $\mathfrak{s}\diamond\mathfrak{r}=constant\ 0$ implies $\mathfrak{r}=\mathfrak{s}$

(5) $\mathfrak{r}\diamond\mathfrak{r}=constant\ 0$

And in Y define a binary relation $\lesssim$ by : $\mathfrak{r}\lesssim\mathfrak{s}\ iff\ \mathfrak{s}\diamond\mathfrak{r}=constant\ 0$, for all $\mathfrak{r},\mathfrak{s}\in Y$.

Thus a KU-g Y hold the conditions :

(1) $\mathfrak{s}\diamond\mathfrak{r}\lesssim\mathfrak{r}$

(2) $constant\ 0\lesssim\mathfrak{r}$

(3) $\mathfrak{r}\lesssim\mathfrak{s}$, $\mathfrak{s}\lesssim\mathfrak{r}$ implies $\mathfrak{r}=\mathfrak{s}$

(4) $(\mathfrak{s}\diamond\mathfrak{t})\diamond(\mathfrak{r}\diamond\mathfrak{t})\lesssim(\mathfrak{r}\diamond\mathfrak{s})$

Theorem $(\mathbf{2.2})[8]$

In any KU-g Y, then the following hold: $\forall\,\mathfrak{r},z,\mathfrak{t}\in Y$

(1) $\mathfrak{r}\lesssim z$ imply $z\diamond\mathfrak{t}\lesssim\mathfrak{r}\diamond\mathfrak{t}$

(2) $\Big(\big((z\diamond\mathfrak{r})\diamond\mathfrak{r}\big)\diamond\mathfrak{r}\Big)=z\diamond\mathfrak{r}$,

(3) $\big((z\diamond\mathfrak{r})\diamond\mathfrak{r}\big)\lesssim z$

(4) $\mathfrak{r}\diamond(z\diamond\mathfrak{t})=z\diamond(\mathfrak{r}\diamond\mathfrak{t})$

Definition $(\mathbf{2.3})[\mathbf{12}]$

If an element "e" of a KU-g Y hold $\mathfrak{r}\lesssim e$ for all $\mathfrak{r}\in Y$, then the element "e" is said to be a unit in Y. A KU-g with unit is denoted by the bounded (or by B.D)

In B.D KU-g Y, we denote $\mathfrak{r}\diamond e$ by $\mathfrak{r}^{\diamond}$ for any $\mathfrak{r}\in Y$.

It is easy to see that $e^{\diamond}=constant\ 0$, $0^{\diamond}=e$.

Lemma $(\mathbf{2.4})[7]$

The unit of B.D KU-g is a unique.

Proposition $(\mathbf{2.5})[7]$

In any B.D KU-g Y, the following are hold:

For any $k,\mathfrak{s}\in Y$,

(1) $k^{\diamond\diamond}\lesssim k$

(2) $k\diamond\mathfrak{s}^{\diamond}=\mathfrak{s}\diamond k^{\diamond}$

(3) $k^{\diamond}\diamond\mathfrak{s}^{\diamond}\lesssim\mathfrak{s}\diamond k$,.

(4) If $\mathfrak{s}\lesssim k$ implies $k^{\diamond}\lesssim\mathfrak{s}^{\diamond}$

(5) $e^{\diamond}\lesssim k$

Definition (2.6)[11] Let $\mathcal{A}$ be a nonempty subset of a KU-g Y. Then $\mathcal{A}$ is said to be a KU-subalgebra of Y, if $\mathfrak{r} \diamond \mathfrak{s} \in \mathcal{A}$, whenever $\mathfrak{r}, \mathfrak{s} \in \mathcal{A}$.

Definition (2.7)[10, 11]
Let $\mathcal{A}$ be a nonempty subset from a KU-g Y. Then $\mathcal{A}$ is said to be an ideal from Y, if
(1) $constant\ 0 \in \mathcal{A}$
(2) $\forall\, \mathfrak{r}, \mathfrak{s} \in Y$, if $\mathfrak{s} \diamond \mathfrak{r} \in \mathcal{A}$ and $\mathfrak{s} \in \mathcal{A}$ imply $\mathfrak{r} \in \mathcal{A}$.

Proposition (2.8)[10] Every ideal in KU-g is KU-subalgebra.

Definition (2.9)[8]
Let $\mathcal{A}$ be a nonempty subset from a KU-g Y. Then $\mathcal{A}$ is said to be a KU-ideal from Y, if
(1) $constant\ 0 \in \mathcal{A}$
(2) $\forall\, \mathfrak{r}, \mathfrak{s}, \mathfrak{t} \in Y$, if $\mathfrak{r} \diamond (\mathfrak{s} \diamond \mathfrak{t}) \in \mathcal{A}$ and $\mathfrak{s} \in \mathcal{A}$ imply $\mathfrak{r} \diamond \mathfrak{t} \in \mathcal{A}$.

Definition (2.10)[7]
Let $\mathcal{A}$ be a non-empty subset in KU-g Y is denoted be complete ideal (briefly, c-ideal), if
(1) $constant\ 0 \in \mathcal{A}$
(2) $\mathfrak{s} \diamond k \in \mathcal{A}$, $\forall\, \mathfrak{s} \in \mathcal{A}$ s.t $\mathfrak{s} \neq constant\ 0$ implies $k \in \mathcal{A}$.

Definition (2.11)[7]
Let Y be B.D KU-g. An element $\mathfrak{r} \in Y$ satisfies $\mathfrak{r}^{\circ\circ} = \mathfrak{r}$, then $\mathfrak{r}$ is called an involution. If every element $\mathfrak{r} \in Y$ is an involution, we call Y is a KU- involutory algebra.

Definition (2.12)[7]
Let $\mathcal{A}$ be a non-empty subset in B.D KU-g Y is called E-ideal if, for all $\mathfrak{r} \in Y$
(1) $constant\ 0 \in \mathcal{A}$
(2) $\mathfrak{r} \diamond k^{\circ} \in \mathcal{A}$ and $k \in \mathcal{A}$ implies $\mathfrak{r}^{\circ} \in \mathcal{A}$.

Definition (2.13)[7]
Let $\mathcal{A}$ be a non-empty subset in B.D KU-g Y is denoted be complete E-ideal (briefly, c-E-ideal), if
(1) $constant\ 0 \in \mathcal{A}$
(2) $\mathfrak{r} \diamond \mathfrak{s}^{\circ} \in \mathcal{A}$, for all $\mathfrak{s} \in \mathcal{A}$ such that $\mathfrak{s} \neq constant\ 0$ implies $\mathfrak{r}^{\circ} \in \mathcal{A}$, $\forall\, \mathfrak{r} \in Y$.

Proposition (2.14)[7] Every c-ideal of B.D KU-g is c-E-ideal .

Proposition (2.15)[7] If $\mathcal{A}$ be c-E-ideal in KU-involutory algebra Y, then $\mathcal{A}$ is c-ideal .

Definition (2.16)[13]
Let Y be set and let φ be a fuzzy set (or fuz. set) in Y is define the mapping $\varphi : Y \longrightarrow [0,1]$.
If φ and ρ be two fuz. subsets in Y , then $\varphi \subseteq \rho \iff \varphi(\mathfrak{r}) \leq \rho(\mathfrak{r})$, for all $\mathfrak{r} \in Y$.

Definition (2.17)[5]
If φ and ρ are two fuz. sets in Y. Then
(1) $(\varphi \cap \rho)(\mathfrak{r}) = min\{\varphi(\mathfrak{r}), \rho(\mathfrak{r})\}$, for all $\mathfrak{r} \in Y$.
(2) $(\varphi \cup \rho)(\mathfrak{r}) = max\{\varphi(\mathfrak{r}), \rho(\mathfrak{r})\}$, for all $\mathfrak{r} \in Y$
$\varphi \cap \rho$ and $\varphi \cup \rho$ are fuz. sets of Y .
In general , if $\{\varphi_\alpha : \alpha \in \lambda\}$ is family of fuz. sets of Y , then
$\bigcap_{\alpha\in\lambda} \varphi_\alpha(\mathfrak{r}) = inf\{\varphi_\alpha(\mathfrak{r}), \alpha \in \lambda\}$, for all $\mathfrak{r} \in Y$ and
$\bigcup_{\alpha\in\lambda} \varphi_\alpha(\mathfrak{r}) = sup\{\varphi_\alpha(\mathfrak{r}), \alpha \in \lambda\}$, for all $\mathfrak{r} \in Y$
Which are also fuz. sets in Y.

Definition (2.18)[8]
Let Y be KU-g. φ be a fuz. set in Y is denoted be a fuz. KU-subalgebra in Y if it hold , $\forall\, k, \mathfrak{s} \in Y$:

(1) $\varphi\,(\,constant\ 0\,) \geq \varphi\,(\,k\,)$
(2) $\varphi\,(\,k \diamond \mathfrak{s}\,) \geq min\{\,\varphi\,(\,k\,),\ \varphi\,(\,\mathfrak{s}\,)\,\}$

Definition (2.19)[8]
Let Y be KU-g . A fuz. set φ of Y is called fuz. KU-ideal in Y if it hold , $\forall\, k, \mathfrak{s}, \mathfrak{t} \in Y$:
(1) $\varphi\,(\,constant\ 0\,) \geq \varphi\,(\,k\,)$
(2) $\varphi\,(\,k \diamond \mathfrak{t}\,) \geq min\{\,\varphi\,(\,k \diamond (\,\mathfrak{s} \diamond \mathfrak{t}\,)\,),\ \varphi\,(\,\mathfrak{s}\,)\}$

Lemma (2.20)[8] Let φ be fuz. KU-ideal in KU-g Y, if $\mathfrak{r} \diamond \mathfrak{s} \lesssim \mathfrak{t}$ is hold, then $\varphi\,(\,\mathfrak{s}\,) \geq min\{\,\varphi\,(\,\mathfrak{r}\,),\ \varphi\,(\,\mathfrak{t}\,)\,\}$

Definition (2.21)[8]
Let φ be fuz. set of a KU-g Y is called fuz. ideal of Y if, $\forall\, \mathfrak{r}, \mathfrak{s} \in Y$
(1) $\varphi\,(\,constant\ 0\,) \geq \varphi\,(\,\mathfrak{r}\,)$
(2) $\varphi\,(\,\mathfrak{r}\,) \geq min\{\,\varphi\,(\,\mathfrak{s} \diamond \mathfrak{r}\,),\ \varphi\,(\,\mathfrak{s}\,)\,\}$

Lemma (2.22)[9]
Let φ be fuz. subset in KU-g Y . Then (1) $\cong$ (2) if :
(1) φ is fuz. ideal
(2) φ is fuz. KU-ideal

Lemma (2.23)[9]
Every fuz. KU-ideal in KU-g Y is fuz. KU-subalgebra .
Remark (2.24)[9] The converse of lemma (2.23) is not true as shown in the example .

Example (2.25)[9]
Consider $Y = \{0,1,2,3,4\}$ be set as shown in the table:

$\diamond$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	0	3	4
2	0	1	0	3	4
3	0	0	0	0	4
4	0	0	0	0	0

Then $(Y, \diamond, constant\ 0)$ is a KU-g .
Define the fuz. set $\varphi : Y \longrightarrow [0,1]$ by :
$$\varphi\,(\,\mathfrak{r}\,) = \begin{cases} 1 & if\ \ \mathfrak{r} = 0,2,3 \\ 0.5 & if\ \ \mathfrak{r} = 1,4 \end{cases}$$
Notice that φ is fuz. KU-subalgebra in Y .

But φ is not fuz. KU-ideal in Y , since
$\varphi\,(0 \diamond 1) = 0.5 \ngeq min\{\,\varphi\,(\,0 \diamond (\,3 \diamond 1\,)\,),\ \varphi\,(3)\,\} = 1$

Proposition (2.26)
Let φ be fuz. KU-subalgebra in a KU-g Y, if $k \diamond \mathfrak{s} \lesssim \mathfrak{t}$ and the inequality $\varphi\,(\,\mathfrak{s}\,) \geq min\{\,\varphi\,(\,k\,),\ \varphi\,(\,\mathfrak{t}\,)\}$ hold , $\forall\, k, \mathfrak{s}, \mathfrak{t} \in Y$. Then φ is fuz. KU-ideal of Y.
Proof:
Let φ be a fuz. KU-subalgebra of Y.
Now, by using Definition (2.18) we get $\varphi\,(\,constant\ 0\,) \geq \varphi\,(\,k\,),\ \forall\, k \in Y$.
Let $k, \mathfrak{s} \in Y$, from Theorem (2.2)(3) we have $(\,\mathfrak{s} \diamond k\,) \diamond k \lesssim \mathfrak{s}$, then $\varphi\,(\,k\,) \geq min\{\ \varphi\,(\,\mathfrak{s}\,),\ \varphi\,(\,\mathfrak{s} \diamond k\,)\ \}$ (By the condition above).
So φ is fuz. ideal of Y , this mean φ is fuz. KU-ideal of Y .

3. The Main Results

We provide definitions in this section : fuz. complete ideal , fuz. E-ideal , fuz. complete E-ideal and study its relationships with fuz. ideal in B.D KU-g .

Definition (3.1)

Let $\mathcal{A}$ be a c-ideal in B.D KU-g Y. Then the fuz. subset $\varphi_{\mathcal{A}} : Y \longrightarrow [0,1]$ is called fuz. complete ideal (briefly, fuz. c-ideal) if , $\forall\, \mathfrak{r} \in Y$ and $\mathfrak{s} \in \mathcal{A}$

(1) $\varphi_{\mathcal{A}}\,(\,constant\ 0\,) \geq \varphi_{\mathcal{A}}\,(\,\mathfrak{r}\,)$
(2) $\varphi_{\mathcal{A}}\,(\,\mathfrak{r}\,) \geq min\{\,\varphi_{\mathcal{A}}\,(\,\mathfrak{s} \diamond \mathfrak{r}\,),\ \varphi_{\mathcal{A}}\,(\,\mathfrak{s}\,)\,\}$

Example (3.2)

Consider the following KU-g Y , [8] with the following table:

⋄	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	3	3
3	0	0	2	0	2
4	0	0	0	0	0

Notice that Y is B.D with unit 4.
A subset $\mathcal{A} = \{0,1,2\}$ is c-ideal of Y. If $\varphi_{\mathcal{A}}$ is fuz. set defined as :

$$\varphi_{\mathcal{A}}\,(\,\mathfrak{r}\,) = \begin{cases} 0.4 & if\ \ \mathfrak{r} = 0,1,4 \\ 0.3 & if\ \ \mathfrak{r} = 2,3 \end{cases}$$

Then $\varphi_{\mathcal{A}}$ is fuz. c-ideal in Y.

While $\rho_{\mathcal{A}}\,(\,\mathfrak{r}\,) = \begin{cases} 0.3 & if\ \ \mathfrak{r} = 0,1,2,3 \\ 0.2 & if\ \ \mathfrak{r} = 4 \end{cases}$

is not fuz. c-ideal in Y , since

$$\rho_{\mathcal{A}}\,(\,4\,) = 0.2 \ngeq min\{\,\rho_{\mathcal{A}}\,(\,2 \diamond 4\,),\ \rho_{\mathcal{A}}\,(\,2\,)\,\} = 0.3$$

Proposition (3.3)

Every fuz. ideal in B.D KU-g is a fuz. c-ideal .

Proof :

Let $\mathcal{A}$ be a c-ideal , $\varphi_{\mathcal{A}}$ be fuz. ideal of B.D KU-g Y, then by Definition (2.21) we have : $\forall\, k, \mathfrak{s} \in Y$

(1) $\varphi_{\mathcal{A}}\,(\,constant\ 0\,) \geq \varphi_{\mathcal{A}}\,(\,k\,)$
(2) $\varphi_{\mathcal{A}}\,(\,k\,) \geq min\{\,\varphi_{\mathcal{A}}\,(\,\mathfrak{s} \diamond k\,),\ \varphi_{\mathcal{A}}\,(\,\mathfrak{s}\,)\,\}$

Since $\mathcal{A} \subseteq Y$, then $\varphi_{\mathcal{A}}\,(\,k\,) \geq min\{\,\varphi_{\mathcal{A}}\,(\,\mathfrak{s} \diamond k\,),\ \varphi_{\mathcal{A}}\,(\,\mathfrak{s}\,)\}$, for every $\mathfrak{s} \in \mathcal{A}$

Thus $\varphi_{\mathcal{A}}$ is fuz. c-ideal of Y .

Remark (3.4)

The conversely of the prop. (3.3) is false as in the next example.

Example (3.5)

In example (3.2) , $\varphi_{\mathcal{A}}$ is fuz. c-ideal in Y (when $\mathcal{A} = \{0,1,2\}$), but it ' s not fuz. ideal , since

$\varphi_{\mathcal{A}}(\,2\,) = 0.3 \ngeq min\{\,\varphi_{\mathcal{A}}(\,4 \diamond 2\,),\ \varphi_{\mathcal{A}}(\,4\,)\,\} = 0.4$

Corollary (3.6)

Let $\mathcal{A}$ be c-ideal in KU-involutory algebra Y. A fuz. subset $\varphi_{\mathcal{A}}$ is fuz. c-ideal if and only if satisfies , $\forall\, k \in Y$ and $\forall\, \mathfrak{s} \in \mathcal{A}$

(1) $\varphi_{\mathcal{A}}\,(\,constant\ 0\,) \geq \varphi_{\mathcal{A}}\,(\,k\,)$
(2) $\varphi_{\mathcal{A}}\,(\,k^{\circ\circ}\,) \geq min\{\,\varphi_{\mathcal{A}}\,(\,\mathfrak{s} \diamond k^{\circ\circ}\,),\ \varphi_{\mathcal{A}}\,(\,\mathfrak{s}\,)\}$

Proof:

By Definition (3.1) and Definition (2.11).

Proposition (3.7)

Let $\mathcal{A}$ be c-ideal in B.D KU-g Y and $\{ \varphi_{\mathcal{A}_i} : i \in \Delta \}$ be family of fuz. c-ideals , then $\bigcap_{i\in\Delta} \varphi_{\mathcal{A}_i}$ is fuz. c-ideal

Proof :

Let $\mathfrak{r} \in Y$, $\mathfrak{s} \in \mathcal{A}$, then

(1) $\varphi_{\mathcal{A}_i}(constant\ 0) \geq \varphi_{\mathcal{A}_i}(\mathfrak{r})$

$inf\{\varphi_{\mathcal{A}_i}(constant\ 0)\} \geq inf\{\varphi_{\mathcal{A}_i}(\mathfrak{r})\}$. So $\bigcap_{i\in\Delta} \varphi_{\mathcal{A}_i}(constant\ 0) \geq \bigcap_{i\in\Delta} \varphi_{\mathcal{A}_i}(\mathfrak{r})$

(2) $\varphi_{\mathcal{A}_i}(\mathfrak{r}) \geq min\{\varphi_{\mathcal{A}_i}(\mathfrak{s} \diamond \mathfrak{r}), \varphi_{\mathcal{A}_i}(\mathfrak{s})\}$, $\forall\, \mathfrak{s} \in \mathcal{A}$

Thus , $inf\{\varphi_{\mathcal{A}_i}(\mathfrak{r})\} \geq inf\{min\{\varphi_{\mathcal{A}_i}(\mathfrak{s} \diamond \mathfrak{r}), \varphi_{\mathcal{A}_i}(\mathfrak{s})\}\}$

$\geq min\{inf\{\varphi_{\mathcal{A}_i}(\mathfrak{s} \diamond \mathfrak{r})\}, inf\{\varphi_{\mathcal{A}_i}(\mathfrak{s})\}\}$

So $\bigcap_{i\in\Delta} \varphi_{\mathcal{A}_i}(\mathfrak{r}) \geq min\{\bigcap_{i\in\Delta} \varphi_{\mathcal{A}_i}(\mathfrak{s} \diamond \mathfrak{r}), \bigcap_{i\in\Delta} \varphi_{\mathcal{A}_i}(\mathfrak{s})\}$, $\forall\, \mathfrak{s} \in \mathcal{A}$

Then $\bigcap_{i\in\Delta} \varphi_{\mathcal{A}_i}$ is fuz. c-ideal of Y .

Remark (3.8) In B.D KU-g. Notice that union of two fuz. c-ideals does not necessary fuz. c-ideal and the following example shows that .

Example (3.9)

Consider $Y = \{0, q, w, h, v, t\}$ and a binary operation $\diamond$ is defined by :

$\diamond$	0	q	w	h	v	t
0	0	q	w	h	v	t
q	0	0	w	h	v	w
w	0	q	0	h	v	q
h	0	0	0	0	v	0
v	0	0	0	0	0	0
t	0	0	0	h	v	0

Then $(Y, \diamond, constant\ 0)$ is a B.D KU-g with unit v [7] . A subset $\mathcal{A} = \{0, q, w, t\}$ is c-ideal of Y. Define fuz. sets $\varphi_{\mathcal{A}}$ and $\rho_{\mathcal{A}}$ from Y into $[0,1]$ by :

$$\varphi_{\mathcal{A}}(\mathfrak{r}) = \begin{cases} 0.6 & if\ \ \mathfrak{r} = 0, w, h, v \\ 0.3 & if\ \ \mathfrak{r} = q, t \end{cases}$$ and

$$\rho_{\mathcal{A}}(\mathfrak{r}) = \begin{cases} 0.7 & if\ \ \mathfrak{r} = 0, q, h, v \\ 0.5 & if\ \ \mathfrak{r} = w, t \end{cases}$$

Then $\varphi_{\mathcal{A}}$ and $\rho_{\mathcal{A}}$ are fuz. c-ideals .

But

$$(\varphi_{\mathcal{A}} \cup \rho_{\mathcal{A}})(\mathfrak{r}) = \begin{cases} 0.7 & if\ \ \mathfrak{r} = 0, q, h, v \\ 0.6 & if\ \ \mathfrak{r} = w \\ 0.5 & if\ \ \mathfrak{r} = t \end{cases}$$

is not fuz. c-ideal , since

$$(\varphi_{\mathcal{A}} \cup \rho_{\mathcal{A}})(t) = 0.5 \ngeq min\{(\varphi_{\mathcal{A}} \cup \rho_{\mathcal{A}})(w \diamond t), (\varphi_{\mathcal{A}} \cup \rho_{\mathcal{A}})(w)\} = 0.6$$

Definition (3.10)

A fuz. subset φ in B.D KU-g Y is called fuz. E-ideal, if

(1) $\varphi(constant\ 0) \geq \varphi(k)$

(2) $\varphi(k^{\circ}) \geq min\{\varphi(k \diamond \mathfrak{s}^{\circ}), \varphi(\mathfrak{s})\}$, for all $k, \mathfrak{s} \in Y$

Note that every fuz. constant in B.D KU-g Y is fuz. E-ideal.

Example (3.11)
Consider $Y = \{0, q, d, h, v\}$ in which the operation $\diamond$ is given by the table:

$\diamond$	0	q	d	h	v
0	0	q	d	h	v
q	0	0	d	h	h
d	0	q	0	h	v
h	0	0	0	0	q
v	0	0	0	0	0

Then $(Y, \diamond, constant\ 0)$ is a B.D KU-g with unit v [7].
If φ and ρ are two fuz. sets defined as :

$$\varphi(\mathfrak{r}) = \begin{cases} 0.8 & if\ \mathfrak{r} = 0, q, h, v \\ 0.6 & if\ \mathfrak{r} = d \end{cases},$$

$$\rho(\mathfrak{r}) = \begin{cases} 0.9 & if\ \mathfrak{r} = 0, d, h, v \\ 0.7 & if\ \mathfrak{r} = q \end{cases}$$

Then φ is fuz. E-ideal in Y .

While ρ is not fuz. E-ideal , since
$\rho(h^\circ) = 0.7 \ngeq min\{\rho(h \diamond v^\circ),\ \rho(v)\} = 0.9$

Proposition (3.12)
Every fuz. ideal in B.D KU-g is fuz. E-ideal

Proof :
Let φ be fuz. ideal in B.D KU-g Y, then by Definition (2.21), we have :
(1) $\varphi(constant\ 0) \geq \varphi(k)$
(2) For all $k, \mathfrak{s} \in Y$, $\varphi(k) \geq min\{\varphi(\mathfrak{s} \diamond k),\ \varphi(\mathfrak{s})\}$.
Then $\varphi(k^\circ) \geq min\{\varphi(\mathfrak{s} \diamond k^\circ),\ \varphi(\mathfrak{s})\}$
$= min\{\varphi(k \diamond \mathfrak{s}^\circ),\ \varphi(\mathfrak{s})\}$,
for all $k, \mathfrak{s} \in Y$ (By Prop. (2.5) (2)).
Thus φ is fuz. E-ideal of Y .

Remark (3.13)
The converse of prop. (3.12) does not hold as in the example .

Example (3.14)
In example (3.11) , φ is fuz. E-ideal of Y , but it's not fuz. ideal in Y , since
$\varphi(d) = 0.6 \ngeq min\{\varphi(h \diamond d),\ \varphi(h)\} = 0.8$

Proposition (3.15)
Every fuz. E-ideal in KU-involutory algebra Y is fuz. ideal .

Proof :
If φ be a fuz. E-ideal in Y, by Definition (3.10) , we have :
(1) $\varphi(constant\ 0) \geq \varphi(\mathfrak{r})$
(2) $\varphi(\mathfrak{r})$ equel to $\varphi(\mathfrak{r}^{\circ\circ}) \geq min\{\varphi(\mathfrak{r}^\circ \diamond \mathfrak{s}^\circ),\ \varphi(\mathfrak{s})\}$
$= min\{\varphi(\mathfrak{s} \diamond \mathfrak{r}^{\circ\circ}),\ \varphi(\mathfrak{s})\}$ (By Prop. (2.5) (2))
$= min\{\varphi(\mathfrak{s} \diamond \mathfrak{r}),\ \varphi(\mathfrak{s})\}$, for all $\mathfrak{r}, \mathfrak{s} \in Y$
(Since Y is a KU-involutory algebra) .
Thus φ is a fuz. ideal of Y.
(3) For any $k, \mathfrak{s} \in Y$, if $k^\circ \lesssim \mathfrak{s}$, then $\varphi(k^\circ) \geq \varphi(\mathfrak{s})$.

Proposition (3.16)
If φ be a fuz. E-ideal of a B.D KU-g Y , then
(1) $k \in Y$, $\varphi(k^\circ) \geq \varphi(e)$
(2) $k \in Y$, $\varphi(k^{\circ\circ}) \geq \varphi(k)$

Proof :
(1) Since φ is fuz. E-ideal , then
$\varphi(k^\circ) \geq min\{\varphi(k \diamond e^\circ),\ \varphi(e)\}$
$= min\{\varphi(k \diamond constant\ 0),\ \varphi(e)\}$
$= min\{\varphi(constant\ 0),\ \varphi(e)\}$
$= \varphi(e)$, $\forall k \in Y$
(2) Since φ is fuz. E-ideal , then for all $k \in Y$

$\varphi(k) = min\{\varphi(constant\ 0), \varphi(k)\} = min\{\varphi(k^\circ \diamond k^\circ), \varphi(k)\} \leq \varphi(k^{\circ\circ})$

(3) If $k^\circ \lesssim \mathfrak{s}$ i.e $\mathfrak{s} \diamond k^\circ = constant\ 0$, then

$\varphi(k^\circ) \geq min\{\varphi(k \diamond \mathfrak{s}^\circ), \varphi(\mathfrak{s})\}$, for any $k, \mathfrak{s} \in Y$ (since φ is fuz. E-ideal)

$= min\{\varphi(\mathfrak{s} \diamond k^\circ), \varphi(\mathfrak{s})\}$ (By Prop. (2.5) (2))

$= min\{\varphi(constant\ 0), \varphi(\mathfrak{s})\} = \varphi(\mathfrak{s})$.

Proposition (3.17)

Let $\{\varphi_\alpha : \alpha \in \Delta\}$ be family of fuz. E-ideals in B.D KU-g Y, then $\bigcap_{\alpha\in\Delta} \varphi_\alpha$ is fuz. E-ideal of Y .

Proof :

(1) since $\varphi_\alpha(constant\ 0) \geq \varphi_\alpha(\mathfrak{r})$, $\forall \alpha \in \Delta$, $\forall \mathfrak{r} \in Y$, then $inf\{\varphi_\alpha(constant\ 0)\} \geq inf\{\varphi_\alpha(\mathfrak{r})\}$.

So $\bigcap_{\alpha\in\Delta} \varphi_\alpha(constant\ 0) \geq \bigcap_{\alpha\in\Delta} \varphi_\alpha(\mathfrak{r})$

(2) let $\mathfrak{r}, \mathfrak{s} \in Y$, since $\varphi_\alpha(\mathfrak{r}^\circ) \geq min\{\varphi_\alpha(\mathfrak{r} \diamond \mathfrak{s}^\circ), \varphi_\alpha(\mathfrak{s})\}$, then

$inf\{\varphi_\alpha(\mathfrak{r}^\circ)\} \geq inf\{min\{\varphi_\alpha(\mathfrak{r} \diamond \mathfrak{s}^\circ), \varphi_\alpha(\mathfrak{s})\}\}$

$\geq min\{inf\{\varphi_\alpha(\mathfrak{r} \diamond \mathfrak{s}^\circ)\}, inf\{\varphi_\alpha(\mathfrak{s})\}\}$

So $\bigcap_{\alpha\in\Delta} \varphi_\alpha(\mathfrak{r}^\circ) \geq min\{\bigcap_{\alpha\in\Delta} \varphi_\alpha(\mathfrak{r} \diamond \mathfrak{s}^\circ), \bigcap_{\alpha\in\Delta} \varphi_\alpha(\mathfrak{s})\}$

Hence $\bigcap_{\alpha\in\Delta} \varphi_\alpha$ is fuz. E-ideal of Y .

Remark (3.18)

Note that the union of two fuz. E-ideals does not necessarily fuz. E-ideal as in the following example .

Example (3.19)

In example (3.2), notice that :

$\varphi(\mathfrak{r}) = \begin{cases} 0.8 & if\ \mathfrak{r} = 0,3 \\ 0.3 & if\ \mathfrak{r} = 1,2,4 \end{cases}$,

$\rho(\mathfrak{r}) = \begin{cases} 0.7 & if\ \mathfrak{r} = 0,1,2 \\ 0.4 & if\ \mathfrak{r} = 3,4 \end{cases}$

are fuz. E-ideals of Y.

But $(\varphi \cup \rho)(\mathfrak{r}) = \begin{cases} 0.8 & if\ \mathfrak{r} = 0,3 \\ 0.7 & if\ \mathfrak{r} = 1,2 \\ 0.4 & if\ \mathfrak{r} = 4 \end{cases}$

is not fuz. E-ideal , since

$(\varphi \cup \rho)(1^\circ) = 0.4 \ngeq min\{(\varphi \cup \rho)(1 \diamond 2^\circ), (\varphi \cup \rho)(2)\} = 0.7$

Definition (3.20)

Let $\mathcal{A}$ be c-E-ideal in B.D KU-g Y. The fuz. subset $\varphi_{\mathcal{A}} : Y \longrightarrow [0,1]$ is called fuz. complete E-ideal (briefly, fuz. c-E-ideal) if ,

(1) $\forall k \in Y$, $\varphi_{\mathcal{A}}(constant\ 0) \geq \varphi_{\mathcal{A}}(k)$

(2) $\forall \mathfrak{s} \in \mathcal{A}$, $\varphi_{\mathcal{A}}(k^\circ) \geq min\{\varphi_{\mathcal{A}}(k \diamond \mathfrak{s}^\circ), \varphi_{\mathcal{A}}(\mathfrak{s})\}$.

Example (3.21)

In example (3.11) , a subset $\mathcal{A} = \{0, q, d\}$ is c-E-ideal in Y .

Let $\varphi_{\mathcal{A}}$ and $\rho_{\mathcal{A}}$ are fuz. sets defined as the following :

$\varphi_{\mathcal{A}}(\mathfrak{r}) = \begin{cases} 0.9 & if\ \mathfrak{r} = 0, d, h \\ 0.6 & if\ \mathfrak{r} = q, v \end{cases}$

and $\rho_{\mathcal{A}}(\mathfrak{r}) = \begin{cases} 0.8 & if\ \mathfrak{r} = 0, q, h \\ 0.5 & if\ \mathfrak{r} = d, v \end{cases}$

Then $\varphi_{\mathcal{A}}$ is fuz. c-E-ideal in Y. While $\rho_{\mathcal{A}}$ is not fuz. c-E-ideal in Y, since

$\rho_{\mathcal{A}}(0^\circ) = 0.5 \ngeq min\{\rho_{\mathcal{A}}(0 \diamond q^\circ), \rho_{\mathcal{A}}(q)\} = 0.8$

Proposition (3.22)

Every fuz. E-ideal in B.D KU-g is fuz. c-E-ideal .

Proof :

Let $\mathcal{A}$ be c-E-ideal in B.D KU-g Y and $\varphi_{\mathcal{A}}$ be fuz. E-ideal of Y , then by Definition (3.10) , we have :

(1) $\varphi_{\mathcal{A}}(\, constant\ 0\,) \geq \varphi_{\mathcal{A}}(\, k\,)$

(2) $\varphi_{\mathcal{A}}(\, k^{\circ}\,) \geq min\{\varphi_{\mathcal{A}}(\, k \diamond \mathfrak{s}^{\circ}\,), \varphi_{\mathcal{A}}(\, \mathfrak{s}\,)\}$, for all $k, \mathfrak{s} \in Y$.

Since $\mathcal{A} \subseteq Y$, then $\varphi_{\mathcal{A}}(\, k^{\circ}\,) \geq min\{\varphi_{\mathcal{A}}(\, k \diamond \mathfrak{s}^{\circ}\,), \varphi_{\mathcal{A}}(\, \mathfrak{s}\,)\}$, $\forall\, \mathfrak{s} \in \mathcal{A}$

Thus $\varphi_{\mathcal{A}}$ is fuz. c-E-ideal in Y .

Remark (3.23)

The converse of prop. (3.22) be not true and the example shows that.

Example (3.24)

In example (3.21) , $\varphi_{\mathcal{A}}$ is fuz. c-E-ideal in Y , but it's not fuz. E-ideal , since

$\varphi_{\mathcal{A}}(\, h^{\circ}\,) = 0.6 \ngeq min\{\, \varphi_{\mathcal{A}}(\, h \diamond h^{\circ}\,), \varphi_{\mathcal{A}}(\, h\,)\} = 0.9$

Corollary (3.25)

Every fuz. ideal in B.D KU-g is fuz. c-E-ideal .

Proof :

By Proposition (3.12) and Proposition (3.22) .

Proposition (3.26)

Every fuz. c-ideal of B.D KU-g is a fuz. c-E-ideal .

Proof :

Let $\varphi_{\mathcal{A}}$ be fuz. c-ideal in B.D KU-g Y & $\mathcal{A}$ is c-ideal of Y.

Then $\mathcal{A}$ is c-E-ideal of Y (By Prop. (2.14)).

Since $\varphi_{\mathcal{A}}$ is fuz. c-ideal of Y, from Definition (3.1) , we have :

(1) $\varphi_{\mathcal{A}}(\, constant\ 0\,) \geq \varphi_{\mathcal{A}}(\, \mathfrak{r}\,)$, $\forall\, \mathfrak{r} \in Y$

(2) $\forall\, \mathfrak{s} \in \mathcal{A}$, $\varphi_{\mathcal{A}}(\, \mathfrak{r}\,) \geq min\{\varphi_{\mathcal{A}}(\, \mathfrak{s} \diamond \mathfrak{r}\,), \varphi_{\mathcal{A}}(\, \mathfrak{s}\,)\}$

Thus $\varphi_{\mathcal{A}}(\, \mathfrak{r}^{\circ}\,) \geq min\{\varphi_{\mathcal{A}}(\, \mathfrak{s}\,), \varphi_{\mathcal{A}}(\, \mathfrak{s} \diamond \mathfrak{r}^{\circ}\,)\,\}$

$= min\{\varphi_{\mathcal{A}}(\, \mathfrak{s}\,), \varphi_{\mathcal{A}}(\, \mathfrak{r} \diamond \mathfrak{s}^{\circ}\,), \}$, for every $\mathfrak{s} \in \mathcal{A}$

So $\varphi_{\mathcal{A}}$ is fuz. c-E-ideal of Y .

Remark (3.27)

In general , not every fuz. c-E-ideal is fuz. c-ideal and the following example shows that.

Example (3.28)

Consider $Y = \{0, q, d, h, v\}$ be set as shown in the following table:

$\diamond$	0	q	d	h	v
0	0	q	d	h	v
q	0	0	d	d	d
d	0	0	0	q	q
h	0	0	0	0	q
v	0	0	0	0	0

Then $(Y, \diamond, constant\ 0)$ is a B.D KU-g with unit v [7].

A subset $\mathcal{A} = \{0, q, h\}$ is c-E-ideal and c-ideal (in the same time) of a B.D KU-g Y.

Define the fuz. set $\varphi_{\mathcal{A}} : Y \longrightarrow [0,1]$ by :

$$\varphi_{\mathcal{A}}(\, \mathfrak{r}\,) = \begin{cases} 0.8 & if\ \ \mathfrak{r} = 0, q, d, v \\ 0.2 & if\ \ \mathfrak{r} = h \end{cases}$$

Then $\varphi_{\mathcal{A}}$ is a fuz. c-E-ideal of Y.

But $\varphi_{\mathcal{A}}$ is not fuz. c-ideal , since

$\varphi_{\mathcal{A}}(\, h\,) = 0.2 \ngeq min\{\varphi_{\mathcal{A}}(\, q \diamond h\,), \varphi_{\mathcal{A}}(\, q\,)\} = 0.8$

Proposition (3.29)

Every fuz. c-E-ideal in a KU-involutory algebra Y is fuz. c-ideal .

Proof :

Let $\varphi_{\mathcal{A}}$ be fuz. c-E-ideal in Y and $\mathcal{A}$ is c-E-ideal of Y.

Then $\mathcal{A}$ is c-ideal of Y (By Prop. (2.15)) .

Since $\varphi_{\mathcal{A}}$ is fuz. c-E-ideal of Y, by Definition (3.20) , we have :

(1) $\forall\, \mathfrak{r} \in Y,\ \varphi_{\mathcal{A}}(\,constant\ 0\,) \geq \varphi_{\mathcal{A}}(\,\mathfrak{r}\,)$

(2) $\forall\, \mathfrak{s} \in \mathcal{A},\ \varphi_{\mathcal{A}}(\,\mathfrak{r}\,) = \varphi_{\mathcal{A}}(\,\mathfrak{r}^{\circ\circ}\,) \geq min\{\,\varphi_{\mathcal{A}}(\,\mathfrak{r}^{\circ} \diamond \mathfrak{s}^{\circ}\,),\ \varphi_{\mathcal{A}}(\,\mathfrak{s}\,)\}$

$= min\{\,\varphi_{\mathcal{A}}(\,\mathfrak{s} \diamond \mathfrak{r}^{\circ\circ}\,),\ \varphi_{\mathcal{A}}(\,\mathfrak{s}\,)\,\}$ (By Prop. (2.5) (2))

$= min\{\,\varphi_{\mathcal{A}}(\,\mathfrak{s} \diamond \mathfrak{r}\,),\ \varphi_{\mathcal{A}}(\,\mathfrak{s}\,)\,\}$

(Since Y is KU-involutory algebra) .

Thus $\varphi_{\mathcal{A}}$ is fuz. c-ideal of Y.

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