[0,1] Truncated Fréchet-Pareto Distributions

Enas Abd Alhafudh Mohammad[1], Sada Faydh Mohammad[2]

Statistics department, Administration and Economics college, Karbala university, Iraq.

Author: ,[1] Enas17stat@gmail.com [2] Sada.faydh.m@gmail.com

Abstract

: *This work, a new continuous distribution built depend on [0,1] truncated Fréchet Pareto ([0,1] TFP) has been introduced and discussed. The cumulative distribution function and properties of the new distribution are obtained under consideration. It is clear that any item fails when it has a pressure to which it is subjected surpasses the conforming strength. Strength, can be noticed as "resistance to failure" in this case. The expected stress is less than techniques for strength if there is a good design practice. We can define the safety factor in expressions of strength and stress as strength/stress. Therefore, this work will derive the [0,1] TFP strength–stress model with more than one parameters. Moreover, this paper will drive the Shannon entropy and Relative entropy and Renyi entropy as well.*

Keywords: [0,1] TFP, Shannon entropy, Stress-strength model, Ranyi entropy, Relative entropy

1. Introduction

H, the distribution we introduced here, can be used with better applicability in other areas. Our guide will be the generality that driven by Eugene et al. [2]. The equation below is the beta G distribution from a quite arbitrary cumulative distribution function (cdf), G(x) [2]:–

$$F(x) = \frac{1}{B(a,b)} \cdot \int_0^{G(x)} (w)^{a-1} (1-w)^{b-1} \, dw \quad \ldots\ldots\ldots\ldots\ldots (1)$$

The supplementary parameters $a > 0$ and $b > 0$ are Asume to a present skewness and to contrast tail weight, also $B(a,b) = \int_0^1 (w)^{a-1} \ (1-w)^{b-1} \ dw$ is the beta function

The attention increased for the class of distributions in equation (1) after 2002 by Eugene et al. [2] as well as Jones [6]. Application of $X = G^{-1}(V)$
to the random variable V following a beta distribution with parameters a and b, in other word $V \sim B(a,b)$, produces X with CDF in equation (1). The beta normal (BN) distribution was defined by Eugene et al. [2]. The definition was introduced by taking G(x) to be the CDF of the normal distribution and then derived couple of its first moments. Gupta and Nadarajah also present general expressions for the moments of the BN distribution [4]. Moreover, a wider analysis of scientific works on this matter is available in Abid and Hassan [1]. Now, let say that we can rewrite (1) as,

$F(X) = I_{G(X)}$ (a,b)

Where $I_y(a,b) = \frac{1}{B\,(a,b)} \int_0^Y w^{a-1}\,(1-w)^{b-1}\,dw$ represent the incomplete beta function ratio. In other word, the cdf of the beta distribution with the parameters a and b Therefore, we can rapid (2) in terms of the well-known hypergeometric function for general a and b, so we can defined them as,

$$F_1(\alpha\,,\beta\,,\gamma\,,X\,) = \sum_{i=0}^{\infty} \frac{(\alpha)_i(\beta)_i}{(\gamma)_i i!}\,X^i$$

Where $(\alpha)_i = \alpha\,(\alpha+1)\ldots..(\alpha+i-1)$ is the ascending factorial . So we get

$$F(X) = \frac{G(X)^a}{a\,B\,(a,b)} \quad , \quad F_1(a\,,1-b\,,a+1;G(X))$$

The characteristics of the cdf, $F(x)$ for any beta G distribution defined from original $G(x)$ n (1), that it might be monitor from the characteristics of the hypergeometric function which are well recognized, see 9.1 in [3]. So, we can write the probability density function (pdf) consistent

To (1) as:–

$$f(x) = \frac{1}{B(a,b)} G(x)^{a-1}\left(1 - G(x)\right)^{b-1} g(x)$$

Where $g(x) = \frac{\partial G(x)}{\partial x}$ is the pdf of the parent distribution

Now, since the pdf and cdf of [0,1] truncated Fréchet distribution are respectively,

$$h(X) = \frac{ab}{e^{-a}} x^{-(b+1)} e^{-ax^{-b}} \qquad 0 < x < 1 \quad \dots\dots\dots\dots\dots\dots. \quad (4)$$

$$H(x) = \frac{1}{e^{-a}} e^{-ax^{-b}} \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (5)$$

Now, Given two absolutely continuous CDF s , H and G, so that h and g are their corresponding pdfs. We suggest a new distribution F by composing H with G, so that $F(X) = H(G(X))$ is a CDF,

$$F(X) = \int_0^{G(X)} \frac{ab}{e^{-a}} t^{-(b+1)} e^{-at^{-b}} dt = \frac{1}{e^{-a}} e^{-aG(X)^{-b}} \dots\dots\dots\dots\dots \quad (6)$$

With pdf,

$$f(x) = \frac{\partial}{\partial X} F(X) = \frac{\partial}{\partial (X)} \frac{e^{-aG(X)^{-b}}}{e^{-a}}$$

$$= \frac{ab}{e^{-a}} e^{-aG(X)^{-b}} \left(G(X)\right)^{-(b+1)} g(x) \quad \dots\dots\dots\dots\dots \qquad (7)$$

With $G(x)$ being a baseline distribution, we introduce (6) and (7) above, a generalized class of distributions. In this work we called it the[0,1]truncated Fréchet Gdistribution. Let we suppose that G is Pareto distribution.

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respectively, then, by applying (6) and (7) above, we get the CDF and PDF of [0,1] TFP random variable as follows,

$$\ldots\ldots\ldots\ldots \quad (8) \qquad F(x) = \frac{1}{e^{-a}}\, e^{-a\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-b}} \qquad 0 < x < \theta$$

$$f(x) = \frac{dF(x)}{dx}$$

$$= \frac{ab}{e^{-a}} e^{-a\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b)}} \left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b+1)} \frac{\theta p^{\theta}}{x^{\theta+1}} \ldots\ldots\ldots\ldots\ldots\ldots \qquad (9)$$

$$a, b, p, \theta > 0 \quad this\ is\ p.d.f$$

prove that f(x) is probability density function

$$\int_0^{\infty} f(x)dx = \int_0^{\infty} \frac{ab\theta p^{\theta}}{e^{-a}}\, e^{-a\left[1-\left(\frac{p}{x}\right)\right]^{-b}} \left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b+1)} dx \ldots\ldots\ldots\ldots.(10)$$

$$= \frac{ab\theta p^{\theta}}{e^{-a}} \int_0^{\infty} x^{-(\theta+1)}\, e^{-a\left[1-\left(\frac{p}{x}\right)\right]^{-b}} \left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b+1)} dx$$

y = $\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b)} \Rightarrow y^{-\frac{1}{b}} = 1-\left(\frac{p}{x}\right)^{\theta} \Rightarrow x = p\left(1-y^{-\frac{1}{b}}\right)^{\frac{1}{\theta}}$

$with \quad dx = -\frac{p}{\theta}\left(1-y^{-\frac{1}{b}}\right)^{-\frac{1}{\theta}-1} \frac{1}{b}\, y^{-\frac{1}{b}-1}\, dy \quad ,then \quad \int_0^{\infty} f(x)dx$

$$= \frac{ab\theta p^{\theta}}{e^{-a}} \int_0^{\infty} e^{-ay} \left(p\left(1-y^{-\frac{1}{b}}\right)^{\frac{1}{\theta}}\right)^{-(\theta+1)} \left(y^{-\frac{1}{b}}\right)^{-(b+1)}$$
$$-\frac{p}{\theta}\left(1-y^{-\frac{1}{b}}\right)^{-\frac{1}{\theta}-1} \frac{1}{b}\, y^{-\frac{1}{b}-1}\, dy \ldots..(11)$$

$$\frac{1}{e^{-a}} \int_0^{\infty} e^{-y}\, dy = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

So it is a p.d.f. Now, for some arbitrary parameters values of pdf and cdf with the result can be driven the central momenGraphs as shown, respectively, in Fig (1) and Fig (2).

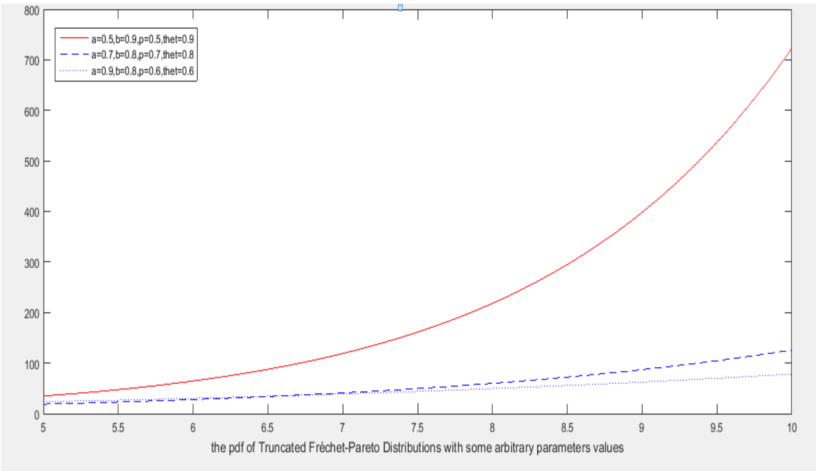

Figure (1) pdf (TFP)

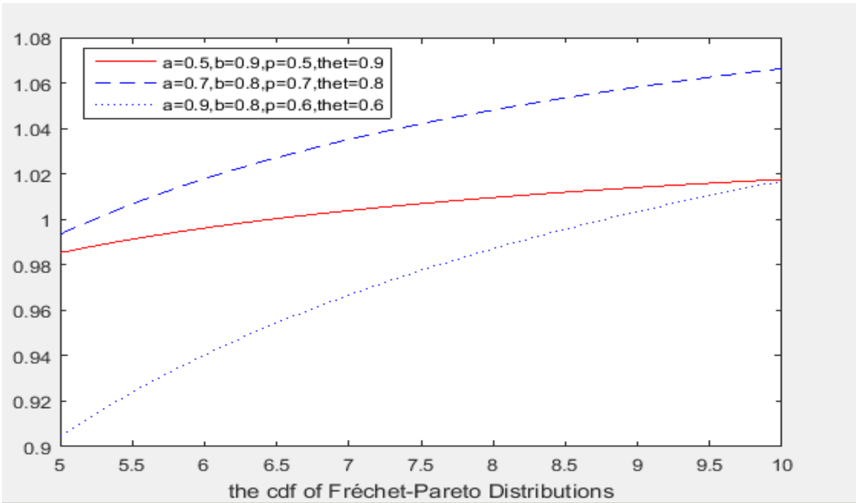

Figure (2) C.D.F (TFP)

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1.3 the rth central moment

$$Ex^r = \int_0^p \frac{ab\theta p^\theta}{e^{-a}} x^{r-\theta-1} e^{-a\left[1-\left(\frac{p}{x}\right)^\theta\right]^{-(b)}} \left[1-\left(\frac{p}{x}\right)^\theta\right]^{-(b+1)} \dots\dots\dots\dots \quad (13)$$

Let $y = \left[1-\left(\frac{p}{x}\right)^\theta\right]^{-(b)} \Rightarrow ,x = p\left(1-y^{\left(-\frac{1}{b}\right)}\right)^{-\frac{1}{\theta}}$

then $dx = -\frac{p}{\theta}\left(1- y^{-\frac{1}{b}}\right)^{-\frac{1}{\theta}-1} \frac{1}{b}\, y^{\left(-\frac{1}{b}-1\right)}\, dy$,

$$Ex^r = \frac{ab\theta p^{\theta}}{e^{-a}}\int_a^{\infty}\left[\mathrm{p}\left(1-y^{\left(-\frac{1}{b}\right)}\right)^{-\frac{1}{\theta}}\right]^{r-\theta-1} e^{-ay}y^{\left(1-\frac{1}{b}\right)}$$

$$-\frac{p}{\theta}\left(1- y^{-\frac{1}{b}}\right)^{-\frac{1}{\theta}-1}\frac{1}{b}\, y^{\left(-\frac{1}{b}-1\right)}\, dy \quad \ldots\ldots\ldots\ldots\ldots\ldots.\ldots\ldots\ldots.(14)$$

$$= \frac{ap^r}{e^{-a}}\int_o^{\infty}\left(1- y^{-\frac{1}{b}}\right)^{-\frac{r}{\theta}} e^{-ay}\, dy \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

by using the Series expansion

$$(1-z)^{-k} = \sum_{j=o}^{\infty}\frac{\Gamma(k+j)}{j!\ \Gamma k}\, z^j \qquad |z|<1 \quad , \qquad k > o$$

$$\left(1- y^{-\frac{1}{b}}\right)^{-\frac{r}{\theta}} = \sum_{j=o}^{\infty}\frac{\Gamma\left(\frac{r}{\theta}+j\right)}{j!\ \Gamma\frac{r}{\theta}}\, y^{-\frac{j}{b}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

$$= \frac{ap^r}{e^{-a}}\sum_{j=o}^{\infty}\frac{\Gamma\left(\frac{r}{\theta}+j\right)}{j!\ \Gamma\frac{r}{\theta}}\int_o^{\infty} y^{-\frac{j}{b}}\ e^{-ay} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(17)$$

Let Z = ay $\Rightarrow y = \frac{z}{a}$ $dy = \frac{dz}{a}$

Hence

$$= \frac{ap^r}{e^{-a}}\sum_{j=o}^{\infty}\frac{\Gamma\left(\frac{r}{\theta}+j\right)}{j!\ \Gamma\frac{r}{\theta}}\int_o^{\infty}\left(\frac{z}{a}\right)^{-\frac{j}{b}} e^{-z}\,\frac{dz}{a} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(18)$$

$$= \frac{a^{-\frac{1}{b}}p^r}{e^{-a}}\sum_{j=o}^{\infty}\frac{\Gamma\left(\frac{r}{\theta}+j\right)}{j!\ \Gamma\frac{r}{\theta}}\,.\Gamma\left(1-\frac{j}{b}\right) \quad \ldots\ldots\ldots\ \ldots\ldots\ldots. \quad (19)$$

.

2.3The Characteristic function is

$Q_x(t) \quad =$

$\sum_{j=o}^{\infty} \frac{(it)^r}{r!} E(\, x^r) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \qquad (20)$

Since $\quad e^{ixt} = \frac{(it)^r}{r!} (\, x^r)$

$$Q_x(t) = \sum_{j=o}^{\infty} \frac{(it)^r}{r!} \cdot \frac{a^{\frac{j}{b}} p^r}{e^{-a}} \sum_{j=o}^{\infty} \frac{\Gamma\left(\frac{r}{\theta}+j\right)}{j!\ \Gamma\left(\frac{r}{\theta}\right)} \ \Gamma\left(1-\frac{j}{b}\right) \quad \ldots\ldots\ldots\ldots.... \quad (21)$$

3.3 mean and variance of the of [0,1] FPD random variable

$\mu = \mathrm{E}(x)$

$\frac{a^{\frac{j}{b}} p}{e^{-a}} \sum_{j=o}^{\infty} \frac{\Gamma\left(\frac{r}{\theta}+j\right)}{j!\,\Gamma\left(\frac{r}{\theta}\right)} \ \Gamma\Big(1 -=$

$\frac{j}{b}\Big) \qquad \ldots\ldots\ldots\ldots..\ldots\ldots\ldots\ldots\ldots.. \qquad (22)$

$= Var(x) = Ex^2 - (Ex)^2$

$\sigma^2 =$

$$\frac{a^{\frac{j}{b}} p^2}{e^{-a}} \sum_{j=o}^{\infty} \frac{\Gamma\left(\frac{2}{\theta}+j\right)}{j!\,\Gamma\left(\frac{2}{\theta}\right)} \Gamma\left(1-\frac{j}{b}\right) - \left(\frac{a^{\frac{j}{b}} p}{e^{-a}} \sum_{j=o}^{\infty} \frac{\Gamma\left(\frac{2}{\theta}+j\right)}{j!\,\Gamma\left(\frac{2}{\theta}\right)} \Gamma\left(1-\frac{j}{b}\right)\right)^2 \ \ldots\ldots\ldots \quad (23)$$

4.3 mode M_o and the median M_e

$$f(x) = \frac{ab\theta p^{\theta}}{e^{-a}}\, x^{-(\theta+1)} e^{-a\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b)}} \left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b+1)}$$

$$f'(x) = \frac{ab\theta p^{\theta}}{e^{-a}} \ (\theta+1)^{-(\theta+2)} e^{-a\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b)}} \left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b+1)} +$$

$$e^{-a\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b)}} \left(1-\left(\frac{p}{x}\right)^{\theta}\right)^{-2(b+1)} - \frac{a^2b^2\theta^2p^{\theta+1}}{e^{-a}}\, x^{-(\theta+1)}\, \frac{1}{x^2} \left(\frac{p}{x}\right)^{\theta-1} -$$

$$\frac{ab\,\theta^2 p^{\theta+1}}{e^{-a}} x^{-(\theta+1)}\, \frac{1}{x^2}\left(\frac{p}{x}\right)^{\theta-1} (b+1) e^{-a\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b)}} \left(1-\left(\frac{p}{x}\right)^{\theta}\right)^{-(b+2)}$$

$$= \ldots \quad (24)$$

$$= \frac{-ab\theta p^{\theta}}{e^{-a}}\,(\theta+1)\,x^{-(\theta+2)}\left(1-\left(\frac{p}{x}\right)^{\theta}\right)^{-(b+2)} e^{-a\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b+1)}}$$

$$+\ \frac{a^2b^2\theta^2p^{2\theta}}{e^{-a}}\ x^{-2(\theta+1)}\left(1-\left(\frac{p}{x}\right)^{\theta}\right)^{-2(b+1)} e^{-a\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b)}}$$

$$-\left(1-\left(\frac{p}{x}\right)^{\theta}\right)^{-(b+2)} e^{-a\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b)}}$$

$$= 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \qquad (25)$$

By extract common factors we get

$$\theta p^{\theta}\left[ab\,x^{-\theta}\left(1-\left(\frac{p}{x}\right)^{\theta}\right)\right]^{-(b+1)} - 1 - \frac{x^{-(\theta+1)}}{\left(1-\left(\frac{p}{x}\right)^{\theta}\right)^{-b}}$$

Last eqution has no closed form therefore one can use numerical methods to solve last equation to get the mode of x

<u>The Median of x is :-</u>

Since $F(X) = \frac{1}{e^{-a}}\, e^{-a\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b)}} = \frac{1}{2}$.

$$a - a\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-b} = liny\frac{1}{2} \qquad \ldots\ldots..\ \ldots\ldots\ldots\ldots\ldots.. \qquad \ldots\ldots(26)$$

$$\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-b} = 1 - \frac{liny\frac{1}{2}}{a} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ \ \ldots\ldots\ldots \qquad (27)$$

$$1-\left(\frac{p}{x}\right)^{\theta} = \left[1-\frac{liny\ \frac{1}{2}}{a}\right]^{-\frac{1}{b}} \Rightarrow \left(\frac{p}{x}\right)^{\theta} = 1-\left[1-\frac{liny\ \frac{1}{2}}{a}\right]^{-\frac{1}{b}}$$

$$\frac{p}{x} = \left\{1-\left[1-\frac{liny\ \frac{1}{2}}{a}\right]^{-\frac{1}{b}}\right\}^{\frac{1}{\theta}} \qquad \Rightarrow \qquad x = p\left\{1-\left[1-\frac{liny\ \frac{1}{2}}{a}\right]^{-\frac{1}{b}}\right\}^{\frac{1}{\theta}}$$

$$= lin\, p - \frac{1}{\theta}\, lin\ 1 - \left[1 - \frac{liny\, \frac{1}{2}}{a}\right]^{-\frac{1}{b}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (28) liny\ x$$

then

$$x = Me = linp - \frac{1}{\theta}\, lin\left\{1 - \left[1 - \frac{liny\, \frac{1}{2}}{a}\right]^{-\frac{1}{b}}\right\} \quad (29)$$

5.3 Cofficient of Skewness of [0, 1] TFP random avarable will be

$$\mathrm{K} = \frac{M_3}{M_2^{3/2}} = \frac{E(x)^3 - 3\,\mu\, E\, x^2 + 2\, m^3}{(\sigma^2)^{3/2}}$$

K=

$$\frac{\left[\frac{a^{\frac{j}{p}} p^3}{e^{-a}} \sum_{j=0}^{\infty} \frac{\Gamma\left(\frac{3}{\theta}+j\right)}{j!\,\Gamma\frac{3}{\theta}}\, \Gamma\left(1-\frac{j}{b}\right) - 3\, \frac{a^{\frac{j}{p}} p}{e^{-a}} \sum_{j=o}^{\infty} \frac{\Gamma\left(\frac{1}{\theta}+j\right)}{j!\,\Gamma\frac{1}{\theta}}\, \Gamma\left(1-\frac{j}{b}\right) + 2\left(\frac{a^{\frac{j}{p}} p}{e^{-a}} \sum_{j=o}^{\infty} \frac{\Gamma(\frac{1}{\theta}+j)}{j!\,\Gamma\frac{1}{\theta}}\, \Gamma\left(1-\frac{j}{b}\right)\right)^3\right]}{\left(\frac{a^{\frac{j}{p}} p^2}{e^{-a}} \sum_{j=o}^{\infty} \frac{\Gamma(\frac{2}{\theta}+j)}{j!\,\Gamma\frac{2}{\theta}}\, \Gamma\left(1-\frac{j}{b}\right) - \frac{a^{\frac{j}{p}} p}{e^{-a}} \sum_{j=o}^{\infty} \frac{\Gamma(\frac{1}{\theta}+j)}{j!\,\Gamma\frac{2}{\theta}}\, \Gamma\left(1-\frac{j}{b}\right)\right)^{3/2}} \ldots\ldots (30)$$

6.3 Cofficient of Kurtosis of [0, 1] TFP random avarable will be

$$\mathrm{Kr} = \frac{\mu_4}{\mu_2^{\,2}} - 3 = \frac{E\,(x)^4 - 4\,\mu\, E\,(x^3) - 6\,\mu^2\, E(x^2) - 3\,\mu^4}{(\sigma^2)^2}$$

$$\left\{\left(\frac{a^{\frac{j}{p}} p^4}{e^{-a}}\right) \sum_{j=o}^{\infty} \frac{\Gamma(\frac{4}{\theta}+j)}{j!\,\Gamma\frac{4}{\theta}}\, \Gamma\left(1 - \frac{j}{b}\right)\ \Gamma\left(\frac{a^{\frac{j}{p}} p^3}{e^{-a}} \sum_{j=0}^{\infty} \frac{\Gamma\left(\frac{3}{\theta}+j\right)}{j!\,\Gamma\frac{3}{\theta}} \sum_{j=0}^{\infty} \Gamma\left(1 - \frac{j}{b}\right)\right)\right\}$$

$$+6\left(\left\{\frac{a^{\frac{j}{p}}p}{e^{-a}}\sum_{j=o}^{\infty}\frac{\Gamma(\frac{1}{\theta}+j)}{j!\,\Gamma\frac{1}{\theta}}\,\Gamma\left(1-\frac{j}{b}\right)\right\}\right)^{2}\left\{\frac{a^{\frac{j}{p}}p^{2}}{e^{-a}}\sum_{j=o}^{\infty}\frac{\Gamma(\frac{2}{\theta}+j)}{j!\,\Gamma\frac{2}{\theta}}\,\Gamma\left(1-\frac{j}{b}\right)\right\}$$

$$-3\left\{\frac{a^{\frac{j}{p}}p^{4}}{e^{-a}}\sum_{j=o}^{\infty}\frac{\Gamma\left(\frac{4}{\theta}+j\right)}{j!\,\Gamma\frac{4}{\theta}}\,\Gamma\left(1-\frac{j}{b}\right)\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{31})$$

4 . Shannan entropies :-

An entropies of a random vardom x is measure of variation of the uncerlainty

Shannan entropies of [0,1] TFP (a,b,β, ϑ) random variable x can found as follows .

$$SE = \int_{-\infty}^{\infty} f(x)lin\left(f(x)\right)dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (32)$$

$$\int_{0}^{\infty} f(x)lin \ \frac{ab\theta p^{\theta}}{e^{-a}}\ x^{-(\theta+1)}e^{-a\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b)}}\left[1-\right.$$

$$Se = \left.\left(\frac{p}{x}\right)^{\theta}\right]^{-(b+1)} \quad dx$$

$$= lin\left[\frac{e^{-a}}{ab\theta p^{\theta}}\right] + (\theta+1)E(lin(x)) + aE\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b)} + (b + 1)E(lin\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b)}$$

………(33).

Now

Let $I_1 = (\theta+1)\,E(\ln(x))$

$$= \frac{ab\theta p^{\theta}}{e^{-a}}(\theta + 1)\int_{0}^{\theta}\ln(x)\ x^{-(\theta+1)}\ e^{-a\left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b)}}\left[1 - \left(\frac{p}{x}\right)^{\theta}\right]^{-(b+1)} dx \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (34)$$

By using equation we get:-

$$e^{-a\left[1-(\frac{\beta}{x})^{\theta}\right]^{-(b)}} = \sum_{i=o}^{\infty} \frac{(-1)^i}{i!} \; a^i \left((1-\left(\frac{p}{x}\right)^{\theta} \;\right)^{-bi}$$

$$Since \;\; e^{-az} = \sum_{m=o}^{\infty} \frac{(-az)^m}{m!} = \sum \frac{(-1)^m}{m!} \, a^m \, z^m$$

$$I_1 = \frac{ab\theta \, p^{\theta}}{e^{-a}} (\theta + 1) \sum \frac{(-1)^i}{i!} a^i \int_o^{\infty} \ln(x) \, x^{-(\theta+1)} \left[1 - (\frac{p}{x})^{\theta}\right]^{-bi} \;\; \left[1 - (\frac{p}{x})^{\theta}\right]^{-(b+1)} dx$$

……………….(35)

$$= \frac{ab\theta \, p^{\theta}}{e^{-a}} \; (\theta + 1) \sum \frac{(-1)^i}{i!} \; a^i \;\; \int_o^{\infty} \ln(x) \;\; x^{-(\theta+1)} \left[1 - (\frac{p}{x})^{\theta}\right]^{-(b(i+1)+1)} \quad dx \qquad \dots\dots\dots \;(36)$$

$$the\ Series\ expansion\;\; (1-z)^{-k} = \sum_{o}^{\infty} \frac{\Gamma\,(k+j)}{i!\;\Gamma(k)} \; (z)^j \qquad |z|, \;\; k > 0$$

$$\left[1 - (\frac{\beta}{x})^{\theta}\right]^{-(b(i+1)+1)} = \sum_{j=o}^{\infty} \frac{\Gamma((b(i+1)+1)+j)}{\Gamma(b(i+1)+1} \; \left[(\frac{p}{x})\right]^{\theta j}$$

$$= \frac{ab\theta \, p^{\theta}}{e^{-a}} \; (\theta + 1) \sum \frac{(-1)^i}{i!} \; a^i \;\; \sum_{j=o}^{\infty} \frac{\Gamma\left((b(i+1)+1)+j\right)}{\Gamma(b(i+1)+1} \int_o^{\infty} \ln(x) \;\; x^{-(\theta+1)} \left[\left(\frac{p}{x}\right)\right]^{\theta j} dx \dots\dots\dots (37)$$

$$\text{Since,} \; \int x^{m} \;\ln(x)\; dx = x^{m+1} \left\{\frac{\ln(x)}{m+1} - \frac{1}{(m+1)^2}\right\} \;, \;\text{then,}$$

$$I_1 = \int_0^1 x^{(-(\theta+1)-\theta j)+1} \left\{ \frac{lin(x)}{(-(-(\theta+1)-\theta j)+1} - \frac{1}{(-(\theta+1)-\theta j)+1)^2} \right\} \ldots\ldots (38)$$

$$= \frac{ab\theta\, p^{\theta}}{e^{-a}} (\theta + 1) \sum \frac{(-1)^i}{i!} a^i \sum_{j=o}^{\infty} \frac{\Gamma\big((b(i+1)+1)+j\big)}{\Gamma(b(i+1)+1} \Big[\frac{lin(1)}{(-(-(\theta+1)-\theta j)+1} - \frac{1}{(-(\theta+1)-\theta j)+1)^2} - 0 \Big] \ldots\ldots (39)$$

$$I_1 = \frac{1}{(-(\theta+1)-\theta j)+1)^2} \quad \ldots\ldots \quad (40)$$

let $I_2 = a \ E\left(1-(\frac{p}{x})^{\theta}\right)^{-b}$

$$= \frac{a^2 b p^{\theta} \theta}{e^{-a}} \int_0^{\infty} x^{-(\theta+1)}\, e^{-a\left(1-(\frac{p}{x})^{\theta}\right)^{-b}} \left(1-\left(\frac{p}{x}\right)^{\theta}\right)^{-b} \left[1-(\frac{\beta}{x})^{\theta}\right]^{-(b+1)} dx \ldots\ldots\ldots (41)$$

Since $\quad e^{-a\left(1-(\frac{p}{x})^{\theta}\right)^{-b}} = \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} a^i \left(1-(\frac{p}{x})^{\theta}\right)^{-b}$

Then $= \frac{a^2 b p^{\theta} \theta}{e^{-a}} \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} a^i \int_0^{\infty} x^{-(\theta+1)} \left[1-(\frac{\beta}{x})^{\theta}\right]^{-(b(i+1)+1} dx$

$$I_{22} = \left[1-(\frac{\beta}{x})^{\theta}\right]^{-(b(i+1)+1)} = \sum_{j=o}^{\infty} \frac{\Gamma((b(i+1)+1)+j)}{j!\,\Gamma(b(i+1)+1} \left[(\frac{p}{x})\right]^{\theta j}$$

$$\text{let } y = \left[1 - (\frac{\beta}{x})^{\theta}\right]^{-b} \Rightarrow x = \left(1 - y^{\frac{-1}{b}}\right)^{-\frac{1}{\theta}} p \Rightarrow dx$$

$$= -\frac{p}{b\theta}\left(1 - y^{-\frac{1}{b}}\right)^{-\frac{1-\theta}{\theta}} y^{-(1+\frac{1}{b})}$$

$$I_2 = \frac{a^2}{e^{-a}} \sum \frac{(-1)^i}{i!} \int_0^{\infty} \left(y^{\frac{-1}{b}}\right)^{-2b-1} \left(1 - y^{-\frac{1}{b}}\right)^{j-1} dy$$

SO $$\boldsymbol{I_2} = \frac{a^2}{e^{-a}} \sum \frac{(-1)^i}{i!} = e^a \frac{\Gamma((-2b+j)}{\Gamma(-2b)\Gamma(j))} \ldots\ldots\ldots\ldots\ldots\ldots \quad (42)$$

Let : $I_3 = (b+1)E(lin(1-\left(\frac{p}{x}\right)^{\theta}$

$$= \int_0^{\infty} lin(1-\left(\frac{p}{x}\right)^{\theta} \frac{ab\theta p^{\theta}}{e^{-a}} x^{-(\theta+1)} e^{-a\left[1-(\frac{\beta}{x})^{\theta}\right]^{-(b)}} \left[1 - (\frac{\beta}{x})^{\theta}\right]^{-(b+1)} dx \ldots\ldots (43)$$

Let $y = a\left[1-(\frac{\beta}{x})^{\theta}\right]^{-(b)}$, $\frac{p}{x} = \left(1-\left(\frac{y}{a}\right)^{-\frac{1}{b}}\right)^{\frac{1}{\theta}}$,

$$x = p\left(1-\left(\frac{y}{a}\right)^{-\frac{1}{b}}\right)^{\frac{1}{\theta}} \qquad dx = -\frac{p}{\theta ab}\left(1-\left(\frac{y}{a}\right)^{-\frac{1}{b}}\right)^{-\frac{(1+\theta)}{\theta}} (\frac{y}{a})^{\frac{-(1+b)}{b}}$$

$$= \frac{ab\theta p^{\theta}}{e^{-a}} \int_0^{\infty} lin\left(\frac{y}{a}\right)^{-\frac{1}{b}} \left[p\left(1-\left(\frac{y}{a}\right)^{-\frac{1}{\theta}}\right)^{-\frac{1}{\theta}}\right]^{-(1+\theta)} e^{-y} \left[\left(\frac{y}{a}\right)^{-\frac{1}{b}}\right]^{-(b+1)} \frac{p}{\theta ab}$$

$$\left(1-\left(\frac{y}{a}\right)^{-\frac{1}{b}}\right)^{-\frac{(1+\theta)}{\theta}} (\frac{y}{a})^{\frac{-(1+b)}{b}} dx \qquad \ldots\ldots\ldots\ldots\ldots\ldots \ \ldots \ (44)$$

SO $I_3 = \frac{(b+1)}{\Gamma(e^{-y}, a\, e^{-y})}$

Then the Shannon entropy is :–

$$SE= in \left[\frac{e^{-a}}{ab\theta p^{\theta}}\right] + \frac{1}{(-(\theta+1)-\theta j)+1)^2} + e^{a}\frac{\Gamma((-2b+j)}{\Gamma\left(-2b)\Gamma(j)\right)} + \frac{(b+1)}{\Gamma(e^{-y}, a\, e^{-y})}$$

4. Stress and Strength:–

$$R = \int_0^{\infty} p(x<y) \quad = \int_0^{\infty} f(x)F_y(x)\,dx$$

$$= \int_0^{\infty} \frac{ab\theta p^{\theta}}{e^{-a}} x^{-(\theta+1)} \left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(b+1)} e^{-a\left[1-(\frac{\beta}{x})^{\theta}\right]^{-(b)}} \frac{1}{e^{-a1}} e^{-a1\left[1-\left(\frac{p1}{x}\right)^{\theta 1}\right]^{-b1}} \quad dx$$

$$\frac{ab\theta p^{\theta}}{e^{-a-a1}} \int_0^{\infty} x^{-(\theta+1)} \left[1-\right.$$

$$= \left.(\frac{\beta}{x})^{\theta}\right]^{-(b+1)} e^{-a\left[1-(\frac{\beta}{x})^{\theta}\right]^{-(b)}} \quad e^{-a1\left[1-\left(\frac{p1}{x}\right)^{\theta 1}\right]^{-b1}} \quad dx \ldots (45)$$

$$\text{Let} \quad e^{-a\left[1-(\frac{\beta}{x})^{\theta}\right]^{-(b)}} = \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} a^i \; e^i \left(1-(\frac{p}{x})^{\theta}\right)^{-bi}$$

$$= \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} a^i \int \left(1-\right.$$

$$\left.(\frac{p}{x})^{\theta}\right)^{(-bi+1)+1} \quad x^{-(\theta+1)} \quad e^{-a1\left[1+\left(\frac{p}{x}\right)^{\theta 1}\right]^{-b1}} \; dx \qquad (46)$$

$$\text{Let} \left[1-\left(\frac{p}{x}\right)^{\theta}\right]^{-(bi+1)+1} = \sum_{j=0}^{\infty} \frac{\Gamma\,(b(i+1)+1)+j}{j!\,\Gamma\, b(i+1)+1} \left[\left(\frac{p}{x}\right)\right]^{\theta j}$$

= =

$$\frac{ab\theta p^{\theta}}{e^{-a}} \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} a^i \sum_{j=0}^{\infty} \frac{\Gamma\,(b(i+1)+1)+j}{j!\,\Gamma\, b(i+1)+1} \int x^{-(\theta+1)} \left[\left(\frac{p}{x}\right)\right]^{\theta j} e^{-a1\left[1+\left(\frac{p}{x}\right)^{\theta 1}\right]^{-b1}} \ldots\ldots\ldots (47)$$

Let : $e^{-a1\left[1+(\frac{p}{x})^{\theta 1}\right]^{-b1}} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} a1^k \left(1-(\frac{p1}{x})^{\theta 1}\right)^{-b1k}$

=

$$= \frac{ab\theta p^{\theta}}{e^{-a1-a}} \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} a^i \sum_{j=0}^{\infty} \frac{\Gamma(b(i+1)+1)+j}{j!\ \Gamma b(i+1)+1} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} a1^k \int_0^{\infty} x^{-(\theta+1)} \left[\left(\frac{p}{x}\right)\right]^{\theta j} \left(1(-(\frac{p1}{x})^{\theta 1}\right)^{-b1k} d$$

For $\left(1-(\frac{p1}{x})^{\theta 1}\right)^{-b1k} = \sum_{m=0}^{\infty} \frac{\Gamma\, b1k+m}{m1\ \Gamma b1k} \left(\frac{p1}{x}\right)^{m\,\theta 1}$

=

$$\frac{ab\theta p^{\theta}}{e^{-a1-a}} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty} \frac{(-1^{i+k})}{i!\ k!} a^{i+1} {a_1}^{k} \frac{\Gamma\, b(i+1)+1)+j}{\Gamma\,(b(i+1)+1} \frac{\Gamma-b1k+m}{m!\ \Gamma b1k} \ldots.(48)$$

$$= \frac{x^{-\theta+m\theta j-m\theta 1}}{-\theta+m\theta j-m\theta 1} then \int_0^{\infty} x^{-(\theta+1)} \left[\left(\frac{p}{x}\right)\right]^{\theta j} \left(\frac{p1}{x}\right)^{m\theta 1}$$

By truncate the integral we get :-

$$\frac{b\theta\, p^{\theta}}{e^{-a1-a}} - \theta + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty} \frac{(-1^{i+k})}{i!\ k!} a^{i+1} {a_1}^{k} \frac{\Gamma\, b(i+1)+1)+j}{\Gamma\,(b(i+1)+1} * \frac{\Gamma-b1k+m}{m!\ \Gamma b1k} \frac{x^{-\theta+m\theta j-m\theta 1}}{-\theta+m\theta j-m\theta 1} \quad \ldots\ldots\ldots\ldots\ldots \quad (49)$$

5. summery Conclusions

In data analysis, statistical distributions are used to represent set(s) data. More than one new distributions have been driven lately to extend a familiar families of distributions. To model real data, the new distributions have more applications than the others. Moreover, in some way, foreword of data modeling, the combining some distributions with each other's.

This work introduced a new family of continuous distributions based on [0,1] truncated Fréchet distribution. [0,1] truncated Fréchet pareto distribution and its properties are showed. Also forms for characteristic function such as, r^{th} raw moment, mean, variance, skewness, kurtosis, mode, median, Shannon entropy function and Relative entropy function proven. This paper deals also with the determination of stress-strength reliability R = P[Y < X] when X (strength) and Y (stress) are two independent .

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