Comparing Different Algorithms for Estimating parameters and Reliability Function of weibull distribution

ReemRazzaq Abdul Hussein

reemrazzak@yahoo.com

University of Information Technology & Communications

Abstract

Today computational techniques play an important role in various fields. In this research was to use the language of Matlab programming to simulate and compare three different algorithms are used to estimate the parameter (α,β) and reliability function of two parameters weibulldistribution.the estimation algorithms are moment estimators , maximumlikeihoodestimatorsand L_moment estimators algorithms which implemented on different sample n.and each algorithms is repeated (R=1000),and the results of estimation for parameters and reliability function are compared using statistical measure mean square error and all result are explained in tables.

Introduction

The weibull probability distribution, like another distribution Frechet, gamma,Farlie-Gumbel-Morgenstern, Clayton, Ali-Mikhail-Haq, Gumbel-Hougaard, Gumbel-Barnett, Nelsen Ten.) are based on models [1][2], that approximately describe the joint probability distribution of a group of variables and it is a power transformation of the exponential distribution, many researcher such as Gastwirth, J.L(1971),Balakrishnan and Kocherlakota (1985)[3]andGroeneveld, R.A. (1986) .have been studied this family of wiebull and explain its skewness and estimating its parameter ,by using maximum likehood, and Method of Moments (MOM) ,[4][5][6],[7],[8],[9],[10].

Key words:

Weibull distribution, Maximum likelihood,L_ moment estimators ,moment estimators .

Definition

the probability density function of two parameter's weillbull's defined by:

$f_T(t) := \frac{\alpha}{\beta}\ (\frac{t}{\beta})\ \alpha\text{-}1 e^{-(\frac{t}{\beta})^{\alpha}}$ .....(1) where t>0

when β is scale parameter and α is shape parameter and the cumulative distribution function(C.D.F) is

$f\ _T(t) := 1 - e^{(\frac{t}{\beta})^{\alpha}}$ .....(2)

while the reliability function R(t)=Pr(T>t)

is defined by $R(t)=F_T(t)=e^{-(t/\beta)\alpha}$.............(3) where (t>0) and (α>β>0)

before estimating two parameters (α,β) by maximum likelihood and moments , we derived a formula forth non central moment about origin and it is found

$\mu`r=E(t^r)=\int_0^{\infty} t^r\ (f^T(t))dt$ .....(4)

which lead to

$\mu`r=E(t\ r)=\ \beta^r\ Gamma(\frac{r+\alpha}{\alpha})$.....(5)

and from equation (5) we define the mean of weibull is :

$E(T)=\mu` = \beta\ Gamma(\frac{r+\alpha}{\alpha})$

and $E(T^2) = \beta^2\ Gamma(\frac{2+\alpha}{\alpha})$

$\alpha^2 = E(T^2) - (E(T))^2$

$= \beta^2\ [Gamma\ (\frac{2+\alpha}{\alpha}) - Gamma^2\ (\frac{1+\alpha}{\alpha})]$

Then the moment estimator of scale parameter β, can be obtained from solving :

m1=E(T)

$= \beta \, Gamma(\frac{1+\alpha}{\alpha})\sum \frac{xi}{n}$

$\hat{\beta}_{mom} = x/gamma(\frac{1+\alpha}{\alpha})$

and the shape parameter (α) is obtained from solving (6)

$E(t^2)=\sum \frac{ti^2}{n}$ …..(6)

$= \beta^2 \; Gamma(\frac{2+\alpha}{\alpha}) = \sum \frac{ti}{n}$ ….(7)

Using $\hat{\beta}_{mom} = x/gamma(\frac{1+\alpha}{\alpha})$ in equation (7), we can find $\hat{\alpha}_{mom}$

The maximum likehood estimators this estimator obtained from maximizing the log of likelihood function obtained from equation (1).

$L=\prod_{i=1}^{n} g(ti)$

$L= \frac{\alpha^n}{\beta^n}\prod_{i=1}^{n} t_I^{\alpha-1}(\beta)^{n(1-\alpha)} e^{\sum-(ti/\beta)}$ ….(8)

$\log L=n \log \alpha-n \log \beta +(\alpha-1)\sum_{i=1}^{n} \log t_i + n(1-\alpha)\log \beta - \sum_{i=1}^{n} t^{\alpha} \beta^{-\alpha}$

$\sigma logL/\sigma\alpha = \frac{n}{\alpha} + \sum_{i=0}^{n} logt_i - n \log \beta - \Sigma (t_i/\beta)^{\alpha}\text{-}(1) \log (t_i/\beta)$

$\hat{\alpha}_{ml} = \frac{n}{\Sigma (ti/\beta)\alpha \log (ti/\beta)\alpha + n \log \beta - \sum_{i=0}^{n} logti}$…..(9)

And

$\frac{\sigma \log L}{\sigma\beta} = \frac{-n}{\beta} + \frac{n(1-\alpha)}{\beta} + \alpha\Sigma t_i\alpha\beta^{-\alpha-1} = n\ \alpha/\ \beta = \alpha/\beta\ \beta^{-\alpha}\Sigma\ t_i^{\alpha}$

$n= \Sigma\ ti^{\alpha}/\beta^{\alpha}$

$Br=\int_0^\infty [1-e^{-(\frac{t}{\beta})\alpha}]^r \alpha/\beta\ (t/\beta)^{\alpha-1} e^{-(\frac{t}{\beta})\alpha}$

$\alpha\int_0^\infty [1-e^{-(\frac{t}{\beta})\alpha}]^r\ (t/\beta)^{\alpha} e^{-(\frac{t}{\beta})\alpha} dt$

since $(a+b)^r=\sum_{i=0}^{r} a\, C_{i=0}^{r} a^{rbr-1}$

$\alpha\int_{i=0}^{r} C_i^r (1)^i (-e^{-(\frac{t}{\beta})\alpha})^{r-i}\ (\frac{t}{\beta})^{\alpha} e^{-(\frac{t}{\beta})\alpha}$

$\alpha\sum_{i=o}^{r} C_i^r (-1)^{r-i} \int_0^\infty (\frac{t}{\beta})^{\alpha} e^{-(\frac{t}{\beta})\alpha-(r-i)(\frac{t}{\beta})} dt$

simplify to

$\beta r=\alpha\sum_{i=o}^{r} C_i^r (-1)^{r-i} \int_0^\infty (\frac{t}{\beta})^{\alpha} e^{-(\frac{t}{\beta})\alpha-(1+r-i)} dt$

$Z=(\frac{t}{\beta})^{\alpha}$

$Z^{\frac{1}{\alpha}}=\frac{t}{\beta} \longrightarrow dt=\frac{1}{\alpha} Z^{\frac{1}{\alpha}-1} dz$

$Br=\alpha$

$=\beta\sum_{i=o}^{r} C_i^r (-1)^{r-i} \int_0^\infty Z^{\frac{1}{\alpha}} e^{-Z(1+r-i)} dz$

$\beta\sum_{i=0}^{r} C_i^r (-1)^{r-i} \frac{1}{(1+r-i)^{\frac{1}{\alpha}+1}}\ gamma(\frac{1}{\alpha}+1)$ ……(13)

there for from equaling equation (13),with (12) , we can obtain the L-moment estimators of α,β as:

$b1=\beta 1\ ,b2=\beta 2$

$\frac{1}{nC_1^{n-1}} \sum_{i=1}^{n} C_1^{i-1} t_{(i)}=\beta\sum_{i=0}^{1} C_i^1 (-1)^{1-i} \frac{gamma\ (\frac{1}{\alpha}+1)}{(2-i)^{\frac{1}{\alpha}+1}}$

$\frac{\sum_{i=1}^{n}(i-1)t(i)}{n(n-1)}=\hat{\beta}_L[C_0^1(-1)^{1-i} \frac{gamma\ (\frac{1}{\alpha}+1)}{2^{\frac{1}{\alpha}+1}} + C_0^0(-1)^0 \frac{gamma((\frac{1}{\alpha}+1)}{1^{\frac{1}{\alpha}+1}}$

$\frac{\sum_{i=1}^{n}(i-1)t(i)}{n(n-1)}=\hat{\beta}_L[\frac{-gamma\ (\frac{1}{\alpha}+1)}{2^{\frac{1}{\alpha}+1}} + \frac{gamma((\frac{1}{\alpha}+1)}{1}$

$\frac{\sum_{i=1}^{n}(i-1)t(i)}{n(n-1)}=\hat{\beta}_L[gamma\ (\frac{1}{\alpha}+1)\ [1-\frac{1}{2^{\frac{1}{\alpha}+1}}]$

$$\hat{\beta}_L = \frac{\frac{\sum_{i=1}^{n}(i-1)\mathrm{t(i)}}{n(n-1)}}{gamma\,(\frac{1}{\alpha}+1)\,[1-\frac{1}{2^{\frac{1}{\alpha}+1}}]} \quad \ldots\ldots\ldots\ldots\ldots\ldots(14)$$

And from B2=b2 where

$$\beta \sum_{i=0}^{2} C_1^2\,(-1)^{2-i}\,\frac{gamma((\frac{1}{\alpha}+1)}{3-i^{(\frac{1}{\alpha}+1)}} = \frac{\sum_{i=1}^{n} C_2^{i-1}\mathrm{t(i)}}{nC_1^{n-1}}$$

$$\beta[C_0^2(-1)^2\ \frac{gamma((\frac{1}{\alpha}+1)}{3^{(\frac{1}{\alpha}+1)}} + C_1^2(-1)^1\ \frac{gamma((\frac{1}{\alpha}+1)}{2^{(\frac{1}{\alpha}+1)}} + C_2^2(-1)^0\ \frac{gamma((\frac{1}{\alpha}+1)}{1^{(\frac{1}{\alpha}+1)}} +$$

$$= \frac{\sum_{i=1}^{n} C_2^{i-1}\mathrm{t(i)}}{\frac{n(n-1)(n-2)}{2}}$$

From solving β2=b2 we obtain the equation which lead to $\hat{\alpha}_{L_{mom}}$

$$\hat{\beta}_{L_{mom}} = \mathrm{Gamma}(\frac{1}{\alpha} + 1)(1 - \frac{1}{2^{\frac{1}{\alpha}}} + \frac{1}{3^{(\frac{1}{\alpha}+1)}})$$

$$= \sum_{i=1}^{n} \frac{(i-1)(i-2)t(i)}{n(n-1)(n-2)}$$

Simulation procedure

This section deals with applying simulation program to find theestimater's of two parameters weibull and reliability of weibull using methods of maximumlikehood and method of Moments and L_Moment.by using matlab(MATRIX LABORATORY)applied different set of sample size and each experiment s repeated R=1000 as show in part of programming code and

The part of programming code simulation as following:

```
B=2;
e=2.5;
n=100;
t=[0.5:0.5:2.5];
Rreal=exp(-(t/B).^e); %% real Reliability
```

```
for R=1:1000
u=rand(n,1);
t1=B*((-log(1-u)).^(1/e));
x=sort(t1);

Bmle(q)=((sum(x.^e))/n).^(1/e);
e10=e;
mle=fsolve(@(e1) SOL(e1,x,Bmle(q),n),e10);
emle(q)=mle;
Rmle(q,:)=exp(-(t./Bmle(q)).^emle(q)); %% mle Reliability

Bmom(q)=(mean(x))/gamma((1+e)/e);
a0=.1;
mom=fsolve(@(a) SOL2(a,x,Bmom(q)),a0);
emom(q)=mom;
Rmom(q,:)=exp(-(t/Bmom(q)).^emom(q));%%% mom Reliability
for k=2:n
s1(k)=k-1;
end
b1=(sum(s1'.*x))/(n*(n-1));
L2=(2*b1)-mean(x);
%%%%%%%%%%%% lmom
Blmom(q)=((mean(x))/(gamma((1+e)/e)));
b0=e;
lmom=fsolve(@(b)reem3(b,Blmom(q),L2),b0);
elmom(q)=lmom;
Rlmom(q,:)=exp(-(t/Blmom(q)).^elmom(q));% lmom Reliability
end
Bhad=[mean(Bmle) mean(Bmom) mean(Blmom)]
ehad=[mean(emle) mean(emom) mean(elmom)]
mseB=[mean((Bmle-B).^2) mean((Bmom-B).^2) mean((Blmom-B).^2)]
msee=[mean((emle-e).^2) mean((emom-e).^2) mean((elmom-e).^2)]
Rreal=Rreal
```

```
mRmle=mean(Rmle)
mRmom=mean(Rmom)
mRlmom=mean(Rlmom)
mseRmle1=((mean(Rmle)-Rreal).^2)
mseRmom1=((mean(Rmom)-Rreal).^2)
mseRlmom1=((mean(Rlmom)-Rreal).^2)
mseRmle=mean((mean(Rmle,1)-Rreal).^2)
mseRmom=mean((mean(Rmom,1)-Rreal).^2)
mseRlmom=mean((mean(Rlmom,1)-Rreal).^2)
```

And all the results are explained in the following tables:

n	method	Experiment1		Experiment 2		Experiment 3	
		$B = 1$	$\alpha = 0.5$	$B = 1.5$	$\alpha = 2$	$B = 2$	$\alpha = 2.5$
15	mle	0.3370	0.0131	0.0373	0.2030	0.0443	0.3353
	mom	0.3662	0.0042	0.0402	0.1811	0.0497	0.2385
	lmom	0.3662	0.0009	0.0402	0.2066	0.0497	0.3973
		mle	lmom	mle	mom	mle	mom
25	mle	0.1804	0.0069	0.0232	0.1053	0.0246	0.1894
	mom	0.1913	0.0027	0.0241	0.0969	0.0285	0.1509
	lmom	0.1913	0.0005	0.0241	0.1141	0.0285	0.2333
		mle	lmom	mle	mom	mle	mom
50	mle	0.0777	0.0030	0.0101	0.0447	0.0120	0.0777
	mom	0.0895	0.0015	0.0114	0.0407	0.0137	0.0705
	lmom	0.0895	0.0002	0.0114	0.0522	0.0137	0.1015
		mle	Lmom	mle	mom	mle	mom
100	mle	0.0425	0.0015	0.0055	0.0210	0.0058	0.0355
	mom	0.0519	0.0009	0.0059	0.0196	0.0068	0.0356
	lmom	0.0519	0.0001	0.0059	0.0258	0.0068	0.0477
best		mle	lmom	mle	mom	mle	mle

Table (1) two parameter of wiebul

Table (2) mean square error

n	method	Experiment1		Experiment 2		Experiment 3	
		$B = 1$	$\alpha = 0.5$	$B = 1.5$	$\alpha = 2$	$B = 2$	$\alpha = 2.5$
15	mle	1.0993	0.5424	1.4859	2.1610	1.9787	2.7256
	mom	1.0402	0.5519	1.4970	2.1526	1.9958	2.6050
	lmom	1.0402	0.5078	1.4970	2.0605	1.9958	2.6077
25	mle	1.0462	0.5239	1.4870	2.1066	1.9864	2.6559
	mom	1.0033	0.5383	1.4953	2.0971	1.9996	2.5533
	lmom	1.0033	0.5056	1.4953	2.0512	1.9996	2.5946
50	mle	1.0169	0.5124	1.4932	2.0446	1.9980	2.5592
	mom	0.9963	0.5246	1.4964	2.0340	2.0020	2.4640
	lmom	0.9963	0.5029	1.4964	2.0172	2.0020	2.5279
100	mle	1.0199	0.5056	1.4995	2.0279	2.0017	2.5355
	mom	1.0134	0.5143	1.5019	2.0170	2.0050	2.4408
	lmom	1.0134	0.5011	1.5019	2.0135	2.0050	2.5176

The second table(2) calculate mean square error ,it is found that the best estimator for (β) is MLE,while for(α) ,the first best estimator is ($\hat{\alpha}_{mom}$)*and the second* estimator is $\hat{\alpha}_{Lmom}$ and finally $\hat{\alpha}_{MLE}$ is best.

In table (3) was found the value of reliability function for first experiment.

n	t_i	Real	mle	mom	lmom
15	0.5	0.4931	0.4909	0.4761	0.4684
	1.0	0.3679	0.3603	0.3432	0.3452
	1.5	0.2938	0.2844	0.2673	0.2745
	2.0	0.2431	0.2334	0.2169	0.2270
	2.5	0.2057	0.1966	0.1808	0.1925
	0.5	0.4931	0.4907	0.4820	0.4759
	1.0	0.3679	0.3622	0.3496	0.3511
	1.5	0.2938	0.2868	0.2731	0.2787
	2.0	0.2431	0.2358	0.2218	0.2299

25	2.5	0.2057	0.1987	0.1848	0.1943
50	0.5	0.4931	0.4915	0.4880	0.4837
	1.0	0.3679	0.3643	0.3577	0.3585
	1.5	0.2938	0.2894	0.2814	0.2853
	2.0	0.2431	0.2384	0.2299	0.2355
	2.5	0.2057	0.2011	0.1924	0.1990
100	0.5	0.4931	0.4939	0.4932	0.4904
	1.0	0.3679	0.3679	0.3650	0.3655
	1.5	0.2938	0.2935	0.2895	0.2918
	2.0	0.2431	0.2426	0.2381	0.2415
	2.5	0.2057	0.2052	0.2005	0.2045

Table (3) the values of Reliability function for first experiment

In table (4) was found the values of reliability function for second experiment

n	t_i	Real	mle	mom	lmom
15	0.5	0.8948	0.8970	0.8970	0.8855
	1.0	0.6412	0.6401	0.6451	0.6351
	1.5	0.3679	0.3547	0.3644	0.3650
	2.0	0.1690	0.1570	0.1633	0.1710
	2.5	0.0622	0.0601	0.0612	0.0694
25	0.5	0.8948	0.8972	0.8968	0.8908
	1.0	0.6412	0.6399	0.6433	0.6381
	1.5	0.3679	0.3566	0.3635	0.3639
	2.0	0.1690	0.1583	0.1631	0.1672
	2.5	0.0622	0.0592	0.0604	0.0648
50	0.5	0.8948	0.8953	0.8942	0.8917
	1.0	0.6412	0.6404	0.6408	0.6388
	1.5	0.3679	0.3631	0.3659	0.3661
	2.0	0.1690	0.1640	0.1668	0.1684
	2.5	0.0622	0.0605	0.0616	0.0634
100	0.5	0.8948	0.8961	0.8950	0.8943
	1.0	0.6412	0.6424	0.6423	0.6419
	1.5	0.3679	0.3667	0.3686	0.3686
	2.0	0.1690	0.1670	0.1693	0.1697
	2.5	0.0622	0.0615	0.0628	0.0633

Table (4) the values of Reliability function for second experiment

In table (5) was found the values of reliability function for third experiment

n	t_i	Real	mle	mom	lmom
15	0.5	0.9692	0.9698	0.9651	0.9620
	1.0	0.8380	0.8421	0.8333	0.8298
	1.5	0.6144	0.6123	0.6120	0.6117
	2.0	0.3679	0.3528	0.3663	0.3676
	2.5	0.1743	0.1596	0.1750	0.1764
25	0.5	0.9692	0.9703	0.9659	0.9660
	1.0	0.8380	0.8428	0.8345	0.8365
	1.5	0.6144	0.6155	0.6137	0.6168
	2.0	0.3679	0.3586	0.3686	0.3693
	2.5	0.1743	0.1628	0.1761	0.1743
50	0.5	0.9692	0.9693	0.9650	0.9670
	1.0	0.8380	0.8398	0.8304	0.8360
	1.5	0.6144	0.6155	0.6097	0.6150
	2.0	0.3679	0.3655	0.3692	0.3695
	2.5	0.1743	0.1712	0.1800	0.1761
100	0.5	0.9692	0.9697	0.9655	0.9684
	1.0	0.8380	0.8401	0.8307	0.8382
	1.5	0.6144	0.6166	0.6100	0.6166
	2.0	0.3679	0.3680	0.3704	0.3706
	2.5	0.1743	0.1733	0.1814	0.1764

Table (5) the values of Reliability function for third experiment

In table (6) shows the mean square error for first experiment.

n	t_i	mle	mom	lmom
15	0.5	0.0462e-004	0.2881 e-003	0.6063 e-003
	1.0	0.5716e-004	0.6079 e-003	0.5141e-003
	1.5	0.8917e-004	0.7028e-003	0.3744 e-003
	2.0	0.9373e-004	0.6864e-003	0.2592e-003
	2.5	0.8347e-004	0.6243e-003	0.1743e-003
25	0.5	0.0581e-004	0.1226e-003	0.2958e-003
	1.0	0.3261e-004	0.3328e-003	0.2824e-003
	1.5	0.4903e-004	0.4300e-003	0.2285 e-003
	2.0	0.5323e-004	0.4528e-003	0.1755e-003
	2.5	0.5005e-004	0.4369e-003	0.1316e-003
	0.5	0.0243 e-004	0.0252 e-003	0.8759 e-004
	1.0	0.1266e-004	0.1044e-003	0.8755 e-004
	1.5	0.1943e-004	0.1534 e-003	0.7350e-004

50	2.0	0.2187e-004	0.1740 e-003	0.5840e-004
	2.5	0.2149e-004	0.1769e-003	0.4526e-004
100	0.5	0.6851e-006	0.0001e-004	0.6918e-005
	1.0	0.0026 e-006	0.0822e-004	0.5842e-005
	1.5	0.1155e-006	0.1846e-004	0.4020e-005
	2.0	0.2570 e-006	0.2475e-004	0.2513e-005
	2.5	0.2978e-006	0.2753e-004	0.1442e-005

Table (6) mean square error for first experiment

In table (7) shows the mean square error for second experiment.

n	t_i	mle	mom	lmom
15	0.5	0.0048e-003	0.0452e-004	0.8767e-004
	1.0	0.0012e-003	0.0452e-004	0.3713e-004
	1.5	0.1749e-003	0.0452e-004	0.0852e-004
	2.0	0.1453e-003	0.0452e-004	0.0380e-004
	2.5	0.0045e-003	0.0452e-004	0.5180e-004
25	0.5	0.0056e-003	0.0392e-004	0.1604e-004
	1.0	0.0016e-003	0.0440e-004	0.0942e-004
	1.5	0.1265e-003	0.1893e-004	0.1619e-004
	2.0	0.1158e-003	0.3476e-004	0.0334e-004
	2.5	0.0089e-003	0.0306e-004	0.0700e-004
50	0.5	0.0019e-004	0.0433e-005	0.9798e-005
	1.0	0.0055e-004	0.0160e-005	0.5664e-005
	1.5	0.2302e-004	0.3871e-005	0.3218e-005
	2.0	0.2538e-004	0.5101e-005	0.0418e-005
	2.5	0.0293e-004	0.0354e-005	0.1545e-005
100	0.5	0.1521e-005	0.0031e-005	0.0297e-005
	1.0	0.1525e-005	0.1273e-005	0.0465e-005
	1.5	0.1451e-005	0.0469e-005	0.0579e-005
	2.0	0.3920e-005	0.0072e-005	0.0436e-005
	2.5	0.0412e-005	0.0353e-005	0.1229e-005

Table (7) mean square error of second experiment

In table (8) shows the mean square error for third experiment.

n	t_i	mle	mom	lmom
15	0.5	0.0003e-003	0.1739e-004	0.5239e-004
	1.0	0.0171e-003	0.2169e-004	0.6666e-004
	1.5	0.0044e-003	0.0575e-004	0.0712e-004
	2.0	0.2278e-003	0.0245e-004	0.0006e-004
	2.5	0.2155e-003	0.0052e-004	0.0458e-004
25	0.5	0.0011e-003	0.1084e-004	0.1054e-004
	1.0	0.0238e-003	0.1221e-004	0.0206e-004
	1.5	0.0014e-003	0.0051e-004	0.0592e-004
	2.0	0.0852e-003	0.0048e-004	0.0208e-004
	2.5	0.1321e-003	0.0307e-004	0.00001e-004
50	0.5	0.0007e-005	0.1818e-004	0.4972e-005
	1.0	0.3322e-005	0.5791e-004	0.3823e-005
	1.5	0.1256e-005	0.2197e-004	0.0392e-005
	2.0	0.5548e-005	0.0173e-004	0.2740e-005
	2.5	0.9935e-005	0.3194e-004	0.3244e-005
100	0.5	0.0187e-005	0.1416e-004	0.0617e-005
	1.0	0.4528e-005	0.1416e-004	0.0043e-005
	1.5	0.5013e-005	0.1416e-004	0.4993e-005
	2.0	0.0026e-005	0.1416e-004	0.7505e-005
	2.5	0.1056e-005	0.1416e-004	0.4539e-005

Table (8) mean square error of third experiment

Model	n	mle	mom	lmom	best
1	15	6.5629e-005	5.8187e-004	3.8565e-004	mom
	25	3.8149e-005	3.5501e-004	2.2276e-004	mle
	50	1.5574e-005	1.2678e-004	7.0458e-005	mle
	100	2.7161e-007	1.5795e-005	4.1470e-006	mle
2	15	6.6119e-005	1.2957e-005	3.7787e-005	mom
	25	5.1661e-005	1.3014e-005	1.0398e-005	lmom
	50	1.0413e-005	1.9838e-006	4.1286e-006	mom
	100	1.7658e-006	4.3936e-007	6.0129e-007	mom
3	15	9.3018e-005	9.5608e-006	2.6162e-005	mom
	25	4.8691e-005	5.4232e-006	4.1197e-006	lmom
	50	4.0136e-006	2.6347e-005	3.0344e-006	lmom
	100	2.1621e-006	2.8548e-005	3.5393e-006	mle

Table (9) MSE for reliability in different method and experiments

From table (9) it is known that the best $\hat{R}_{mom}$is best with (41.666%) and then $\hat{R}_{MLE}$ with (33.333%) and finally $\hat{R}_{Lmom}$(25%).

Conclusion

in this paper have been used matlab for writing a program to compare three different algorithms are used to estimate the parameter (α,β) and reliability function of two parameters weibull distribution. the estimation algorithms are moment estimators , maximum likelihood estimators and L_moment estimators algorithms which implemented on different sample n and the results of estimation for parameters and reliability function are compared using MSE for parametersand it is found that the best estimator for (β) is MLE, while for(α) ,the first best estimator is ($\hat{\alpha}_{mom}$)$and\ the\ second$ estimator is $\hat{\alpha}_{Lmom}$ and finally $\hat{\alpha}_{MLE}$ is best while the MSE for reliability it is known that the best $\hat{R}_{mom}$is best with (41.666%) and then $\hat{R}_{MLE}$ with (33.333%) and finally $\hat{R}_{Lmom}$(25%).

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