Comparing Estimators of Scale and Reliability Function of Frechet two Parameters Distribution

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Abstract

This paper deals with comparing different estimator of scale parameter (θ) and reliability function [R(t)] of the two parameters Frechet distribution considering the shape parameter (λ) is known, using different methods, these methods are least square, L – moment, proposed Bayes estimator and maximum likelihood method. The comparison has been done through simulation using different sample size $(n = 30,60,90,120,150)$, and different set of initial values $(\lambda = 2,3,4, \theta = 0.5,1,1.5, c = 1,2)$, each experiment replicated $(R = 500)$, the comparison has been done using mean square error, all results explained in tables.

Keywords: Frechet two parameters, Scale parameter, Shape parameter, MSE, r^{th} Moment.

1. Introduction

The probability distribution of Gumbel and Frechet are widely used for frequency analysis of recorded meteorological data such as rainfall, temperature, wind speed, time to failure of certain produced items. Landwehr etal. (1979) discussed MLM to obtain quality estimator in sample, also Phien (1987) studied that MLM is the most efficient method since it have smallest possible variance, Gautam (2003) and Vivekanandan (2012) estimates the Frechet Distribution This distribution is a special case of the general distribution (extreme value distribution), it has been studied by Maurice Frechet (1927), also Fisher and Tippet (1928), and Gumbel (1958). Applications of the Frechet distribution in various fields given in Harlow (2002) showed that it is an important distribution for modeling the statistical behavior of materials properties for a variety of engineering applications. Nadarajah and Kotz (2008) discussed the sociological models based on Frechet random variables. Further, Zaharim et al. (2009) applied Frechet distribution for analyzing the wind speed data. Mubarak (2011) studied the Frechet progressive type-II censored data with binomial removals.

The probability distribution function $(p.d.f)$ of the random variable (x), which follow two parameters Frechet distribution (θ, λ) which are the scale and shape parameters as;

$$f(x;\theta,\lambda) = \lambda\theta^{\lambda}x^{-(\lambda+1)}e^{-\left(\frac{\theta}{x}\right)^{\lambda}} \qquad x,\theta,\lambda > 0 \qquad (1)$$

The cumulative probability function is;

$$F(x) = pr(X \le x) = \int_0^x f(u)du \ = e^{-\left(\frac{\theta}{x}\right)^{\lambda}} \qquad x, \theta, \lambda > 0 \tag{2}$$

the reliability function is;

$$R(x) = 1 - e^{-\left(\frac{\theta}{x}\right)^{\lambda}} \tag{3}$$

The formula of r^{th} moments about origin is;

$$\mu_r' = E(x^r) = \int_0^{\infty} x^r f(x; \theta, \lambda)dx = \theta^r \, \Gamma\left(\frac{\lambda - r}{\lambda}\right) \tag{4}$$

Then, the moment estimator are obtained from (4) as;

$$\overline{x} = \hat{\theta}_{mom} \, \Gamma\left(\frac{\lambda - 1}{\lambda}\right) \tag{5}$$

$$\hat{\theta}_{mom} = \frac{\overline{x}}{\Gamma\left(\frac{\lambda-1}{\lambda}\right)} \tag{6}$$

2. Methods of estimation

2.1 Maximum Likelihood Method

The estimator obtained from maximizing the logarithm of likelihood function which is defined by;

$$L(x_1, x_2, \dots x_n, \lambda, \theta) = \prod_{i=1}^{n} f(x_i; \lambda, \theta) = \lambda^n \theta^{n\lambda} \prod_{i=1}^{n} x_i^{-(\lambda+1)} e^{-\sum_{i=1}^{n}\left(\frac{\theta}{x_i}\right)^{\lambda}}$$

Then;

$$\ln L = n \ln \lambda + n\lambda \ln \theta - (\lambda + 1) \sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n} \left(\frac{\theta}{x_i}\right)^{\lambda}$$

$$\frac{\partial \ln L}{\partial \theta} = \frac{n\lambda}{\theta} - \lambda \theta^{\lambda - 1} \sum_{i=1}^{n} x_i^{-\lambda}$$

$$put \ \frac{\partial \log L}{\partial \theta} = 0$$

$$\hat{\theta}_{MLE} = \left[\frac{n}{\sum_{i=1}^{n} x_i^{-\lambda}}\right]^{\frac{1}{\lambda}} \tag{7}$$

Which is an implicit function of (λ) Where (λ) is known parameter.

2.2 Least Square Estimator

This method is well known in statistics and mathematics, it depend on linear relation between two variables, one is called dependent variable (y_i), and second is independent variable (x_i), to apply it on two parameters Frechet, we apply the following;

Let;

$$F(x) = e^{-\left(\frac{\theta}{x}\right)^{\lambda}}$$

Be $c.d.f$ of two parameters Frechet, then;

$$\ln F(x) = -\left(\frac{\theta}{x}\right)^{\lambda}$$

$$-\ln F(x) = \left(\frac{\theta}{x}\right)^{\lambda}$$

$$\ln F^{-1}(x) = \left(\frac{\theta}{x}\right)^{\lambda}$$

$$\ln[\ln F^{-1}(x)] = \lambda(\ln\theta - \ln x_{i)}$$

$$y_i = \lambda\ln\theta - \lambda\ln x_i$$

$$\overline{y} = \frac{1}{n}\sum_{i=1}^{n} \ln\left[\ln\frac{1}{F(x_i)}\right] \quad wher\ F(x_i) = \frac{i}{n+1}$$

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} \ln(x_i)$$

$$y_i = \ln\left[\ln\frac{1}{F(x_i)}\right] = \ln\left[\ln\left(\frac{n+1}{i}\right)\right]$$

We estimate (θ) by OLS as follows, and observation [$x_i = \ln(x_i)$], then;

$$\hat{\theta}_{OLS} = e^{\left(\overline{y} - \frac{\overline{x}}{\lambda}\right)} \tag{8}$$

2.3 Estimation using L - Moment

The estimation by this method depends on equating the formula of population moment (β_r) as;

$$\beta_r = \int_0^{\infty} x\, F^r(x) f(x) dx \tag{9}$$

With sample moment (b_r);

$$b_r = \frac{1}{n\, C_r^{n-1}} \sum_{i=1}^{n} C_r^{i-1}\ x_{(i)} \qquad (10)$$

$$\beta_r = \int_0^\infty x\, e^{\left[-\left(\frac{\theta}{x}\right)^{\lambda}\right]^r} \lambda\theta^\lambda x^{-(\lambda+1)} e^{-\left(\frac{\theta}{x}\right)^{\lambda}} dx$$

$$\beta_r = \lambda \int_0^\infty \left(\frac{\theta}{x}\right)^{\lambda}\ e^{-\left(\frac{\theta}{x}\right)^{\lambda r}} e^{-\left(\frac{\theta}{x}\right)^{\lambda}} dx$$

Let;

$$\frac{\theta}{x} = y \quad \rightarrow \quad x = \frac{\theta}{y} \qquad dx = -\ \frac{\theta}{y^2}\ dy$$

$$\beta_r = \lambda \int_0^\infty y^\lambda\ e^{-ry^\lambda} e^{-y^\lambda} \frac{\theta}{y^2}\ dy$$

$$= \theta\lambda \int_0^\infty y^{\lambda-2}\ \ e^{-(r+1)y^\lambda}\ dy$$

Let;

$$z = (r+1)y^\lambda \quad \rightarrow \quad \left(\frac{z}{r+1}\right)^{\frac{1}{\lambda}} = y$$

$$y = (z)^{\frac{1}{\lambda}} \left(\frac{1}{r+1}\right)^{\frac{1}{\lambda}} \qquad dy = \frac{1}{\lambda}(z)^{\frac{1}{\lambda}-1} \left(\frac{1}{r+1}\right)^{\frac{1}{\lambda}}\ dz$$

$$\beta_r = \theta\lambda \int_0^\infty \left[(z)^{\frac{1}{\lambda}}\right]^{\lambda-2}\ e^{-z}\ \frac{1}{\lambda}(z)^{\frac{1}{\lambda}-1} \left(\frac{1}{r+1}\right)^{\frac{1}{\lambda}}\ dz$$

$$= \theta \left(\frac{1}{r+1}\right)^{\frac{1}{\lambda}} \int_0^\infty e^{-z}\ (z)^{1-\frac{2}{\lambda}+\frac{1}{\lambda}-1}\ dz$$

$$= \frac{\theta}{(r+1)^{\frac{1}{\lambda}}} \int_0^\infty e^{-z}\ (z)^{-\frac{1}{\lambda}}\, dz$$

$$= \frac{\theta}{(r+1)^{\frac{1}{\lambda}}} \Gamma\left(1 - \frac{1}{\lambda}\right) \qquad (11)$$

Then;

$$\beta_r = b_r \qquad (12)$$

$$\beta_1 = \frac{\theta}{2^{\frac{1}{\lambda}}} \Gamma\left(1 - \frac{1}{\lambda}\right) \qquad (13)$$

$$b_1 = \frac{1}{n(n-1)} \sum_{i=1}^{n} (i-1)x_{(i)}$$

Where $x_{(1)} \le x_{(2)} \le x_{(3)} \le \cdots \le x_{(n)}$ are ordered observation of sample values $x_1, x_2, \dots, x_n$, from solve $[\beta_1 = b_1]$ and $[\beta_2 = b_2]$ we obtain the estimator by L – moment method, since here we have only one parameter, we apply equation (13).

2.4 Bayes Estimator

Bayes Estimator of scale parameter (θ) ,this Method consider (θ) as random variable have prior distribution $\pi(\theta)$ defined by:

$$\pi(\theta) = k\theta^c \qquad \theta > 0$$

$$Constant \qquad c\,,k > 0$$

According to this we find the posterior distribution

$$g(\theta|x) \propto L(x;\lambda,\theta)\pi(\theta)$$

$$Now\ f(x) = \int L(x;\lambda,\theta)\pi(\theta)\, d\theta$$

Then applying certain steps to find the posterior distribution, which is equal to:

$$g(\theta|x) = \frac{\boldsymbol{\lambda}\, \boldsymbol{\theta}^{c+n\lambda} \boldsymbol{T}^{\frac{c+1}{\lambda}+n} \boldsymbol{e}^{-\Sigma\left(\frac{\theta}{x_i}\right)^{\lambda}}}{\boldsymbol{\Gamma}(\frac{\boldsymbol{c}+\boldsymbol{1}}{\boldsymbol{\lambda}} + \boldsymbol{n})}$$

Which is a p.d.f since

$$\int_{\forall\theta} g(\theta|x)\, d\theta = 1$$

Then according to squared error loss function

$$L = E(\hat{\theta} - \theta)^2$$

We find that the Bayes estimator for parameter θ is

$$\hat{\theta}_{Bayes} = \frac{\Gamma(\frac{c+2}{\lambda}+n)}{T^{\lambda}\, \Gamma(\frac{c+1}{\lambda}+n)}$$

$$where \qquad T = \sum_{i=1}^{n} \left(\frac{1}{x_i}\right)^{\lambda}$$

3. Simulation

The estimator using different sample size $(n = 30,60,90,120,150)$, and the initial values for (λ) and (θ) are;

λ	2	3	4
θ	0.5	1	1.5
C	1	2	

The replicate of each experiment (R=500), the results ($\hat{\theta}$) then [$\hat{R}(t_i)$] are compared using statistical measure (MSE), and applying method of least square, L – moment, Moment, and Maximum Likelihood, all results explained in tables.

Estimation of Scale Parameters ($\boldsymbol{\theta}$) when ($\boldsymbol{\lambda}$) is known as follows;

Table (1): Estimation of Scale Parameters ($\boldsymbol{\theta}$) and mean square error ($\lambda = 2,\ \theta = 0.5,\ c=2$)

n		LS	$L-Mom$	$Bayes$	ML	$Best$
30	$\hat{\theta}$	0.376632	0.360632	0.510876	0.506671	
	$MSE(\hat{\theta})$	0.017145	0.051068	0.002325	0.002338	Bayes
60	$\hat{\theta}$	0.370461	0.356793	0.505568	0.503474	
	$MSE(\hat{\theta})$	0.017736	0.030325	0.001095	0.001082	ML
90	$\hat{\theta}$	0.368723	0.352914	0.503598	0.502205	
	$MSE(\hat{\theta})$	0.017922	0.026921	0.000764	0.000752	ML
120	$\hat{\theta}$	0.368398	0.349229	0.502118	0.501075	
	$MSE(\hat{\theta})$	0.017757	0.025902	0.000545	0.000539	ML
150	$\hat{\theta}$	0.365971	0.351353	0.502069	0.501234	
	$MSE(\hat{\theta})$	0.01843	0.025367	0.000448	0.000443	ML

Table (2): Estimation of Scale Parameters ($\boldsymbol{\theta}$) and mean square error
($\lambda = 3, \theta = 0.5,\ c = 2$)

n		LS	$L-Mom$	$Bayes$	ML	$Best$
30	$\hat{\theta}$	0.467257	0.395318	0.504973	0.504037	
	$MSE(\hat{\theta})$	0.001216	0.013808	0.001057	0.001045	ML
60	$\hat{\theta}$	0.459966	0.395585	0.502056	0.501591	
	$MSE(\hat{\theta})$	0.00168	0.012408	0.000509	0.000506	ML
90	$\hat{\theta}$	0.456919	0.395337	0.501303	0.500994	
	$MSE(\hat{\theta})$	0.00192	0.012425	0.000325	0.000324	ML
120	$\hat{\theta}$	0.454651	0.397026	0.500932	0.5007	
	$MSE(\hat{\theta})$	0.002095	0.011373	0.000241	0.00024	ML
150	$\hat{\theta}$	0.453827	0.396122	0.501248	0.501062	
	$MSE(\hat{\theta})$	0.002161	0.011384	0.000188	0.000189	Bayes

Table (3): Estimation of Scale Parameters ($\boldsymbol{\theta}$) and mean square error($\lambda = 4$, $\theta = 0.5,\ c = 2$)

n		LS	$L-Mom$	$Bayes$	ML	$Best$
30	$\hat{\theta}$	0.501556	0.421859	0.503478	0.503473	
	$MSE(\hat{\theta})$	4.31E-05	0.007669	0.000563	0.000564	Ls
60	$\hat{\theta}$	0.494008	0.420777	0.501695	0.501694	
	$MSE(\hat{\theta})$	5.15E-05	0.006937	0.000253	0.000253	LS

90	$\hat{\theta}$	0.490912	0.419503	0.500203	0.500203	
	MSE($\hat{\theta}$)	9.29E-05	0.006905	0.000179	0.000179	LS
120	$\hat{\theta}$	0.489019	0.420614	0.500444	0.500444	
	$MSE(\hat{\theta})$	1.30E-04	0.006752	0.00012	0.00012	LS
150	$\hat{\theta}$	0.365394	0.354234	0.501529	0.500695	
	$MSE(\hat{\theta})$	1.86E-02	0.02458	0.000442	0.000439	LS

<u>Table (4):</u> Estimation of Scale Parameters ($\boldsymbol{\theta}$) and mean square error ($\lambda = 2, \theta = 1$,c=2)

n		$\boldsymbol{LS}$	$\boldsymbol{L-Mom}$	$\boldsymbol{Bayes}$	$\boldsymbol{ML}$	$Best$
30	$\hat{\theta}$	0.247964	0.70249	1.020799	1.012397	
	$MSE(\hat{\theta})$	5.68E-01	0.143085	0.008996	0.008577	LS
60	$\hat{\theta}$	0.240603	0.71059	1.011521	1.007332	
	$MSE(\hat{\theta})$	5.78E-01	0.118318	0.004451	0.004337	ML
90	$\hat{\theta}$	0.238428	0.703446	1.006155	1.003372	
	MSE($\hat{\theta}$)	5.81E-01	0.102626	0.002765	0.002724	ML
120	$\hat{\theta}$	0.238121	0.701012	1.00558	1.003492	
	$MSE(\hat{\theta})$	5.81E-01	0.100811	0.00204	0.002013	ML
150	$\hat{\theta}$	0.236656	0.702827	1.004516	1.002846	
	$MSE(\hat{\theta})$	5.83E-01	0.097626	0.001692	0.001674	ML

<u>Table (5):</u>Estimation of Scale Parameters ($\boldsymbol{\theta}$) and mean square error($\lambda = 3 , \theta = 1, c = 2$)

n		$\boldsymbol{LS}$	$\boldsymbol{L-Mom}$	$\boldsymbol{Bayes}$	$\boldsymbol{ML}$	$Best$
30	$\hat{\theta}$	0.372851	0.792894	1.009851	1.00798	
	$MSE(\hat{\theta})$	0.393674	0.055429	0.003972	0.003981	Bayes
60	$\hat{\theta}$	0.367172	0.79479	1.002659	1.00173	
	$MSE(\hat{\theta})$	0.40065	0.048435	0.001792	0.001783	ML
90	$\hat{\theta}$	0.363727	0.79714	1.00343	1.002811	
	MSE($\hat{\theta}$)	0.40498	0.045928	0.001329	0.001322	ML
120	$\hat{\theta}$	0.363356	0.79346	1.001384	1.00092	
	$MSE(\hat{\theta})$	0.405406	0.045636	0.001002	0.001012	Bayes
150	$\hat{\theta}$	0.362001	0.793867	1.001816	1.001445	
	$MSE(\hat{\theta})$	0.407114	0.044858	0.000751	0.000749	ML

<u>Table (6):</u>Estimation of Scale Parameters ($\boldsymbol{\theta}$) and Mean square error ($\lambda = 4$, $\theta = 1, \ c = 2$)

n		LS	$L-Mom$	$Bayes$	ML	$Best$
30	$\hat{\theta}$	0.430436	0.84139	1.005236	1.005227	
	$MSE(\hat{\theta})$	3.25E-01	0.030507	0.002209	0.002219	Bayes
60	$\hat{\theta}$	0.424277	0.83779	1.002349	1.002347	
	$MSE(\hat{\theta})$	3.32E-01	0.029234	0.001068	0.001069	Bayes
90	$\hat{\theta}$	0.421215	0.839739	1.001753	1.001752	
	$MSE(\hat{\theta})$	3.35E-01	0.027605	0.000717	0.000716	ML
120	$\hat{\theta}$	0.419678	0.839229	1.001182	1.001181	
	$MSE(\hat{\theta})$	3.37E-01	0.027143	0.000498	0.000497	ML
150	$\hat{\theta}$	0.418768	0.839136	1.00049	1.00049	
	$MSE(\hat{\theta})$	3.38E-01	0.026966	0.000414	0.000413	ML

<u>Table (7):</u> Estimation of Scale Parameters ($\boldsymbol{\theta}$) and Mean square error ($\lambda = 2, \ \theta = 1.5, \ c = 1$)

n		LS	$L-Mom$	$Bayes$	ML	$Best$
30	$\hat{\theta}$	0.164262	1.086442	1.532439	1.526067	
	$MSE(\hat{\theta})$	1.79E+00	1.079284	0.022238	0.02169	ML
60	$\hat{\theta}$	0.156358	1.062928	1.510098	1.506956	
	$MSE(\hat{\theta})$	1.81E+00	0.310369	0.010326	0.01023	ML
90	$\hat{\theta}$	0.155192	1.052071	1.507966	1.505874	
	$MSE(\hat{\theta})$	1.81E+00	0.230109	0.006125	0.00608	ML
120	$\hat{\theta}$	0.154372	1.05465	1.50421	1.502644	
	$MSE(\hat{\theta})$	1.81E+00	0.236126	0.004446	0.004426	ML
150	$\hat{\theta}$	0.15389	1.051368	1.50366	1.502408	
	$MSE(\hat{\theta})$	1.81E+00	0.23051	0.003965	0.003951	ML

<u>Table (8):</u>Estimation of Scale Parameters ($\boldsymbol{\theta}$) and mean square error ($\lambda = 4, \ \theta = 1.5, c = 1$)

n		LS	$L-Mom$	$Bayes$	ML	$Best$
30	$\hat{\theta}$	0.369938	1.260877	1.505293	1.506849	
	$MSE(\hat{\theta})$	1.28E+00	0.069902	0.005293	0.005322	Bayes
60	$\hat{\theta}$	0.36347	1.26317	1.502937	1.503717	
	$MSE(\hat{\theta})$	1.29E+00	0.062522	0.002369	0.002377	**Bayes**
90	$\hat{\theta}$	0.361354	1.261728	1.502491	1.503011	
	$MSE(\hat{\theta})$	1.30E+00	0.060659	0.001546	0.00155	Bayes
120	$\hat{\theta}$	0.359962	1.260681	1.50211	1.502501	
	$MSE(\hat{\theta})$	1.30E+00	0.060208	0.001142	0.001144	Bayes
150	$\hat{\theta}$	0.358939	1.260793	1.502814	1.503127	
	$MSE(\hat{\theta})$	1.30E+00	0.059568	0.000908	0.00091	Bayes

Estimators of Reliability function as follows;

<u>Table (9):</u> Reliability estimator when $(\lambda = 2,\ \theta = 0.5,\ \ c = 2)$

n	t	Real	$\hat{R}_{LS}$	$MSE(\widehat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\widehat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(\widehat{R}_{Bayes})$	$\hat{R}_{Mle}$	$MSE(\widehat{R}_{Mle})$	Best
30	0.5	0.632121	0.433465	0.044113	0.38947	0.081289	0.64548	4.89E-03	0.639514	4.77E-03	MLE
	1	0.221199	0.134762	0.008094	0.121002	0.016021	0.228296	0.001458	0.225025	0.001389	MLE
	1.5	0.105161	0.061838	0.002012	0.060309	0.006873	0.10988	0.000385	0.108184	0.000362	MLE
	2	0.060587	0.035699	0.000658	0.032752	0.001567	0.063968	1.44E-04	0.062954	1.34E-04	MLE
	2.5	0.039211	0.022731	0.000292	0.023775	0.002133	0.041226	6.37E-05	0.040564	5.96E-05	MLE
60	0.5	0.632121	0.423343	0.046371	0.390445	0.073727	0.637372	2.30E-03	0.634346	1.40E-03	MLE
	1	0.221199	0.129407	0.008744	0.120181	0.013913	0.225034	0.000677	0.223403	0.000659	MLE
	1.5	0.105161	0.060055	0.002112	0.055267	0.003354	0.107294	0.000178	0.106457	0.000172	MLE
	2	0.060587	0.034	0.000729	0.032033	0.001104	0.062178	6.50E-05	0.061681	6.27E-05	MLE
	2.5	0.039211	0.021863	0.000312	0.02148	0.000676	0.040225	2.78E-05	0.039899	2.68E-05	MLE
90	0.5	0.632121	0.421358	0.04616	0.386915	0.07054	0.634364	1.52E-03	0.632338	1.52E-03	MLE
	1	0.221199	0.127854	0.008938	0.116602	0.012699	0.224292	0.000424	0.223204	0.000416	MLE
	1.5	0.105161	0.058891	0.002203	0.055772	0.003793	0.106591	0.000104	0.106035	0.000102	MLE
	2	0.060587	0.033674	0.000741	0.030949	0.001058	0.061314	3.81E-05	0.060986	3.73E-05	MLE
	2.5	0.039211	0.021622	0.000317	0.020489	0.000479	0.039871	1.87E-05	0.039656	1.83E-05	MLE
120	0.5	0.632121	0.415776	0.048573	0.393586	0.066564	0.633933	1.24E-03	0.632411	1.23E-03	MLE
	1	0.221199	0.126513	0.00916	0.118701	0.012672	0.223981	0.000337	0.223165	0.000331	MLE
	1.5	0.105161	0.057808	0.002289	0.056272	0.003342	0.106298	8.74E-05	0.105882	8.60E-05	MLE
	2	0.060587	0.03311	0.000769	0.031789	0.001018	0.061388	3.05E-05	0.061141	3.00E-05	MLE
	2.5	0.039211	0.021547	0.000317	0.019789	0.00044	0.039613	1.32E-05	0.039452	1.29E-05	MLE
150	0.5	0.632121	0.414541	0.048847	0.39015	0.066604	0.633021	9.06E-04	0.631801	9.05E-04	MLE
	1	0.221199	0.125987	0.009219	0.118765	0.012293	0.22327	0.000263	0.222618	0.00026	MLE
	1.5	0.105161	0.057984	0.00226	0.0545	0.002896	0.106496	6.80E-05	0.106162	6.68E-05	MLE
	2	0.060587	0.032958	0.000776	0.031709	0.001076	0.060961	2.41E-05	0.060765	2.38E-05	MLE
	2.5	0.039211	0.021272	0.000327	0.020036	0.000434	0.039592	1.08E-05	0.039463	1.07E-05	MLE

Table (10): Reliability estimator when $(\lambda = 3,\ \theta = 0.5,\ \ c = 2)$

n	t_i	$Real$	$\hat{R}_{LS}$	$MSE(\widehat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\widehat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(\widehat{R}_{Bayes})$	$\hat{R}_{Mle}$	$MSE(\widehat{R}_{Mle})$	Best
30	0.5	0.632121	0.556308	0.00655	0.39854	0.068923	0.640333	4.53E-03	0.638321	4.50E-03	**MLE**
	1	0.117503	0.097101	0.000468	0.064512	0.004356	0.120769	0.000444	0.120142	0.000436	**MLE**
	1.5	0.03636	0.029757	4.91E-05	0.01965	0.000417	0.03772	5.09E-05	0.037515	4.98E-05	**MLE**
	2	0.015504	0.012693	8.79E-06	0.008152	7.09E-05	0.016159	9.75E-06	0.01607	9.54E-06	**MLE**
	2.5	0.007968	0.006523	2.32E-06	0.004176	1.80E-05	0.008244	2.54E-06	0.008199	2.49E-06	**MLE**
60	0.5	0.632121	0.539482	0.009006	0.395915	0.062911	0.636553	2.36E-03	0.635539	2.35E-03	**MLE**
	1	0.117503	0.092549	0.000648	0.0625	0.003512	0.119236	0.000227	0.118926	0.000225	**MLE**
	1.5	0.03636	0.028306	6.74E-05	0.018974	0.000346	0.037236	2.52E-05	0.037134	2.49E-05	**MLE**
	2	0.015504	0.012071	1.22E-05	0.008014	6.35E-05	0.015804	4.46E-06	0.01576	4.41E-06	**MLE**
	2.5	0.007968	0.006215	3.19E-06	0.004072	1.70E-05	0.008114	1.23E-06	0.008092	1.21E-06	**MLE**
90	0.5	0.632121	0.533205	0.010074	0.3914	0.06303	0.634694	1.59E-03	0.634016	1.59E-03	**MLE**
	1	0.117503	0.09073	0.000732	0.061237	0.003352	0.118535	0.000135	0.118329	0.000134	**MLE**
	1.5	0.03636	0.02782	7.46E-05	0.018659	0.000338	0.036976	1.58E-05	0.036909	1.57E-05	**MLE**
	2	0.015504	0.011852	1.36E-05	0.007837	6.25E-05	0.015631	2.70E-06	0.015602	2.69E-06	**MLE**
	2.5	0.007968	0.006092	3.61E-06	0.004081	2.35E-05	0.008061	7.70E-07	0.008046	7.65E-07	**MLE**
120	0.5	0.632121	0.528441	0.010969	0.395493	0.059937	0.635445	1.14E-03	0.634936	1.13E-03	**MLE**
	1	0.117503	0.089844	0.000779	0.061062	0.003352	0.118624	0.0001	0.11847	9.95E-05	**MLE**
	1.5	0.03636	0.027536	7.91E-05	0.018365	0.000341	0.036623	1.08E-05	0.036573	1.07E-05	**MLE**
	2	0.015504	0.011673	1.49E-05	0.007932	6.04E-05	0.015749	2.39E-06	0.015727	2.37E-06	**MLE**
	2.5	0.007968	0.006	3.93E-06	0.004057	1.61E-05	0.008035	5.58E-07	0.008024	5.55E-07	**MLE**
150	0.5	0.632121	0.525701	0.011495	0.396298	0.059086	0.635978	9.39E-04	0.63557	9.36E-04	**MLE**
	1	0.117503	0.089155	0.000814	0.061764	0.003291	0.117922	8.56E-05	0.1178	8.53E-05	**MLE**
	1.5	0.03636	0.027289	8.32E-05	0.01853	0.000331	0.036711	9.16E-06	0.036671	9.11E-06	**MLE**
	2	0.015504	0.011605	1.54E-05	0.007916	6.13E-05	0.015666	1.64E-06	0.015649	1.63E-06	**MLE**
	2.5	0.007968	0.005967	4.05E-06	0.004016	1.62E-05	0.008045	4.39E-07	0.008036	4.37E-07	**MLE**

<u>Table (11):</u> Reliability estimator when $(\lambda = 4\ ,\ \theta = 0.5\ ,\ c = 2)$

n	t_i	$Real$	$\hat{R}_{LS}$	$MSE(\widehat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\widehat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(\widehat{R}_{Bayes})$	$\hat{R}_{Mle}$	$MSE(\widehat{R}_{Mle})$	Best
30	0.5	0.632121	0.637985	0.000326	0.391223	0.067972	0.639348	0.004721	0.639336	0.004621	MLE
	1	0.060587	0.061539	8.50E-06	0.032059	0.000975	0.063299	1.49E-04	0.063297	1.48E-04	MLE
	1.5	0.01227	0.012475	3.68E-07	0.006421	4.08E-05	0.01257	5.16E-06	0.012569	5.15E-06	MLE
	2	0.003899	0.003962	3.80E-08	0.002053	4.02E-06	0.004053	6.33E-07	0.004053	6.32E-07	MLE
	2.5	0.001599	0.00162	6.97E-09	0.000856	6.92E-07	0.001665	1.05E-07	0.001665	1.04E-07	MLE
60	0.5	0.632121	0.614304	0.000472	0.392887	0.062893	0.635537	0.002206	0.635534	0.002106	MLE
	1	0.060587	0.05781	1.09E-05	0.031457	0.000915	0.06179	6.62E-05	0.061789	6.61E-05	MLE
	1.5	0.01227	0.011698	4.62E-07	0.006302	3.82E-05	0.012515	2.62E-06	0.012515	2.61E-06	MLE
	2	0.003899	0.003714	4.77E-08	0.002002	3.84E-06	0.003961	2.62E-07	0.003961	2.61E-07	MLE
	2.5	0.001599	0.001524	7.92E-09	0.000821	6.53E-07	0.00163	4.42E-08	0.00163	4.41E-08	MLE
90	0.5	0.632121	0.605102	0.000829	0.391161	0.062047	0.63469	0.001488	0.634689	0.001388	MLE
	1	0.060587	0.056378	1.97E-05	0.031249	0.000901	0.060951	3.71E-05	0.060951	3.70E-05	MLE
	1.5	0.01227	0.011408	8.32E-07	0.006192	3.85E-05	0.012415	1.85E-06	0.012415	1.84E-06	MLE
	2	0.003899	0.003615	9.07E-08	0.002009	3.76E-06	0.003948	1.71E-07	0.003948	1.70E-07	MLE
	2.5	0.001599	0.001486	1.42E-08	0.000804	6.59E-07	0.001608	3.02E-08	0.001608	3.01E-08	MLE
120	0.5	0.632121	0.59974	0.001126	0.392628	0.06053	0.633521	0.001155	0.63352	0.001055	MLE
	1	0.060587	0.055582	2.67E-05	0.031136	0.000901	0.061123	3.04E-05	0.061123	3.034E-05	MLE
	1.5	0.01227	0.011241	1.13E-06	0.006228	3.79E-05	0.012371	1.28E-06	0.012371	1.27E-06	MLE
	2	0.003899	0.003565	1.18E-07	0.001983	3.81E-06	0.003934	1.21E-07	0.003934	1.20E-07	MLE
	2.5	0.001599	0.001463	1.97E-08	0.000815	6.78E-07	0.001609	2.02E-08	0.001609	2.01E-08	MLE
150	0.5	0.632121	0.595825	0.001375	0.39425	0.058786	0.634199	0.000887	0.634198	0.000877	MLE
	1	0.060587	0.055074	3.17E-05	0.031069	0.000897	0.060904	2.31E-05	0.060904	2.30E-05	MLE
	1.5	0.01227	0.011124	1.37E-06	0.006239	3.74E-05	0.012371	9.93E-07	0.012371	9.92E-07	MLE
	2	0.003899	0.003537	1.37E-07	0.001967	3.85E-06	0.003903	1.02E-07	0.003903	1.01E-07	MLE
	2.5	0.001599	0.001448	2.35E-08	0.000809	6.40E-07	0.001612	1.69E-08	0.001612	1.68E-08	MLE

Table (12): Reliability estimator when $(\lambda = 2,\ \theta = 1\ ,\ c = 2)$

n	t_i	$Real$	$\hat{R}_{LS}$	$MSE(\hat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\hat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bayes})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	$Best$
30	0.5	0.981684	0.219374	0.585654	0.820053	0.039134	0.981001	0.000168	0.979797	0.000188	Bayes
	1	0.632121	0.061782	3.26E-01	0.380058	0.083688	0.642127	4.21E-03	0.636145	4.13E-03	MLE
	1.5	0.35882	0.027426	1.10E-01	0.206	3.68E-02	0.372965	3.30E-03	0.36819	3.13E-03	MLE
	2	0.221199	0.015647	4.23E-02	0.122815	1.62E-02	0.231286	1.55E-03	0.22798	1.46E-03	MLE
	2.5	0.147856	0.010122	1.90E-02	0.080857	8.42E-03	0.154324	7.12E-04	0.152003	6.69E-04	MLE
60	0.5	0.981684	0.210307	0.597705	0.833267	0.03003	0.981535	8.14E-05	0.980934	8.60E-05	Bayes
	1	0.632121	0.057829	3.30E-01	0.386637	0.074135	0.639205	2.29E-03	0.636178	2.26E-03	MLE
	1.5	0.35882	0.026327	1.11E-01	0.196313	3.19E-02	0.364747	1.38E-03	0.362369	1.35E-03	MLE
	2	0.221199	0.014681	4.27E-02	0.123216	1.53E-02	0.226453	7.11E-04	0.224813	6.88E-04	MLE
	2.5	0.147856	0.009456	1.92E-02	0.080772	6.92E-03	0.1514	3.43E-04	0.150249	3.32E-04	MLE
90	0.5	0.981684	0.20695	0.602075	0.840121	0.025858	0.981096	5.44E-05	0.980685	5.68E-05	Bayes
	1	0.632121	0.055844	3.32E-01	0.39071	0.067486	0.637116	1.38E-03	0.635088	1.37E-03	MLE
	1.5	0.35882	0.02553	1.11E-01	0.200128	3.09E-02	0.363529	9.41E-04	0.361943	9.23E-04	MLE
	2	0.221199	0.014395	4.28E-02	0.119227	1.26E-02	0.224107	4.18E-04	0.22302	4.10E-04	MLE
	2.5	0.147856	0.009182	1.92E-02	0.077773	5.75E-03	0.149788	2.09E-04	0.149026	2.05E-04	MLE
120	0.5	0.981684	0.202511	0.608554	0.849082	0.022068	0.981749	4.09E-05	0.981448	4.19E-05	Bayes
	1	0.632121	0.055249	3.33E-01	0.390358	0.067724	0.636011	1.09E-03	0.634487	1.08E-03	MLE
	1.5	0.35882	0.024984	1.11E-01	0.200875	2.97E-02	0.361383	7.87E-04	0.360196	7.79E-04	MLE
	2	0.221199	0.014117	4.29E-02	0.119323	1.25E-02	0.223406	3.14E-04	0.222592	3.09E-04	MLE
	2.5	0.147856	0.009131	1.92E-02	0.076915	5.68E-03	0.149565	1.54E-04	0.148993	1.51E-04	MLE
150	0.5	0.981684	0.202846	0.607812	0.847306	0.021639	0.981088	3.46E-05	0.98084	3.56E-05	MLE
	1	0.632121	0.055208	3.33E-01	0.390436	0.065992	0.633482	9.09E-04	0.632262	9.07E-04	MLE
	1.5	0.35882	0.024724	1.12E-01	0.200325	2.83E-02	0.360309	5.42E-04	0.35936	5.38E-04	MLE
	2	0.221199	0.01404	4.29E-02	0.119387	1.31E-02	0.223626	2.90E-04	0.222973	2.86E-04	MLE
	2.5	0.147856	0.009007	1.93E-02	0.077084	5.50E-03	0.149243	1.28E-04	0.148786	1.26E-04	MLE

Table (13): Reliability estimator when $(\lambda = 3,\ \theta = 1,\ c = 2)$

n	t_i	$Real$	$\hat{R}_{LS}$	$MSE(\hat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\hat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bayes})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	$Best$
30	0.5	0.999665	0.3425	0.433663	0.963761	0.002624	0.999362	9.98E-07	0.999338	1.07E-06	Bayes
	1	0.632121	0.050856	0.337921	0.393945	0.069609	0.645819	0.00473	0.64381	0.004682	MLE
	1.5	0.256433	0.015435	0.058085	0.140067	0.01665	0.264338	0.001749	0.26309	0.001718	MLE
	2	0.117503	0.006497	0.012323	0.063049	0.003624	0.122662	5.35E-04	0.122026	5.24E-04	MLE
	2.5	0.061995	0.003337	0.003441	0.033219	0.001234	0.064428	1.45E-04	0.064082	1.42E-04	MLE
60	0.5	0.999665	0.326167	0.454503	0.973132	0.001254	0.999549	2.22E-07	0.999539	2.32E-07	MLE
	1	0.632121	0.04839	0.340771	0.393219	0.065018	0.635863	0.00233	0.634849	0.002324	MLE
	1.5	0.256433	0.014704	0.058435	0.138464	0.015965	0.258944	0.000831	0.258329	0.000825	MLE
	2	0.117503	0.006199	0.012389	0.061459	0.003423	0.119852	2.29E-04	0.11954	2.27E-04	MLE
	2.5	0.061995	0.003175	0.00346	0.032537	0.000989	0.062767	6.03E-05	0.062599	5.98E-05	Bayes
90	0.5	0.999665	0.321625	0.460318	0.975374	0.000905	0.999575	1.26E-07	0.999569	1.30E-07	Bayes
	1	0.632121	0.047233	0.342111	0.395735	0.061219	0.635508	0.001379	0.63483	0.001375	MLE
	1.5	0.256433	0.014297	0.058631	0.138644	0.014803	0.258821	0.000606	0.258411	0.000602	MLE
	2	0.117503	0.006046	0.012423	0.062221	0.003512	0.119674	1.52E-04	0.119466	1.51E-04	MLE
	2.5	0.061995	0.003084	0.003471	0.03259	0.00096	0.062831	4.51E-05	0.062718	4.47E-05	MLE
120	0.5	0.999665	0.318412	0.464548	0.976907	0.000753	0.999605	8.72E-08	0.999601	8.93E-08	Bayes
	1	0.632121	0.0468	0.342613	0.395181	0.060306	0.632753	0.001096	0.632244	0.001095	MLE
	1.5	0.256433	0.014071	0.058741	0.140478	0.014454	0.257787	0.000392	0.25748	0.000391	MLE
	2	0.117503	0.005995	0.012434	0.061252	0.003325	0.11787	1.09E-04	0.117717	1.08E-04	MLE
	2.5	0.061995	0.003072	0.003472	0.03189	0.000955	0.062408	3.34E-05	0.062324	3.33E-05	MLE
150	0.5	0.999665	0.317061	0.466299	0.977513	0.000686	0.99961	6.27E-08	0.999606	6.40E-08	Bayes
	1	0.632121	0.046489	0.342975	0.393747	0.060178	0.63296	0.000953	0.632552	0.000952	MLE
	1.5	0.256433	0.013984	0.058782	0.13939	0.014325	0.258321	0.000345	0.258075	0.000344	MLE
	2	0.117503	0.005901	0.012455	0.0616	0.003267	0.118554	7.87E-05	0.118431	7.83E-05	MLE
	2.5	0.061995	0.003036	0.003476	0.031849	0.000945	0.062444	2.48E-05	0.062377	2.47E-05	MLE

Table (14): Reliability estimator when ($\lambda = 4\ ,\ \theta = 1, c = 2$)

n	t_i	$Real$	$\hat{R}_{LS}$	$MSE(\hat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\hat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bayes})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	Best
30	0.5	1	0.424605	0.331931	0.998054	1.73E-05	0.999999	5.78E-11	0.999999	5.77E-11	MLE
	1	0.632121	0.033974	3.58E-01	0.395768	0.067242	0.637514	4.21E-03	0.637502	4.20E-03	MLE
	1.5	0.179245	0.006792	2.97E-02	0.096924	8.17E-03	0.18422	9.84E-04	0.184214	9.83E-04	MLE
	2	0.060587	0.002158	3.41E-03	0.032235	9.56E-04	0.062948	1.37E-04	0.062945	1.36E-04	MLE
	2.5	0.025275	0.000881	5.95E-04	0.013341	1.69E-04	0.026123	2.48E-05	0.026122	2.47E-05	MLE
60	0.5	1	0.403436	0.356286	0.998987	3.56E-06	1	1.43E-12	1	1.42E-12	MLE
	1	0.632121	0.031773	3.60E-01	0.396362	0.061497	0.636092	2.23E-03	0.636088	2.22E-03	MLE
	1.5	0.179245	0.006376	2.99E-02	0.095793	7.62E-03	0.181973	4.69E-04	0.181971	4.68E-04	MLE
	2	0.060587	0.002015	3.43E-03	0.031803	9.06E-04	0.061677	5.97E-05	0.061677	5.96E-05	MLE
	2.5	0.025275	0.000831	5.98E-04	0.012847	1.67E-04	0.025665	1.12E-05	0.025665	1.11E-05	MLE
90	0.5	1	0.395117	0.366189	0.999211	1.89E-06	1	5.85E-13	1	5.84E-13	MLE
	1	0.632121	0.031016	3.61E-01	0.394628	0.059996	0.631864	1.52E-03	0.631862	1.51E-03	MLE
	1.5	0.179245	0.00619	2.99E-02	0.096054	7.31E-03	0.180595	2.92E-04	0.180594	2.91E-04	MLE
	2	0.060587	0.001962	3.44E-03	0.031316	9.00E-04	0.061129	3.94E-05	0.061129	3.93E-05	MLE
	2.5	0.025275	0.000806	5.99E-04	0.012855	1.62E-04	0.025554	7.49E-06	0.025554	7.48E-06	MLE
120	0.5	1	0.391368	0.370615	0.999347	1.08E-06	1	2.02E-13	1	2.01E-13	MLE
	1	0.632121	0.030565	3.62E-01	0.393585	0.05972	0.633426	1.14E-03	0.633425	1.13E-03	MLE
	1.5	0.179245	0.006097	3.00E-02	0.095296	7.31E-03	0.180688	2.29E-04	0.180688	2.28E-04	MLE
	2	0.060587	0.001938	3.44E-03	0.031019	9.05E-04	0.061025	2.98E-05	0.061025	2.97E-05	MLE
	2.5	0.025275	0.000794	5.99E-04	0.012756	1.62E-04	0.025465	5.04E-06	0.025465	5.03E-06	MLE
150	0.5	1	0.388202	0.374476	0.999379	9.43E-07	1	9.06E-14	1	9.05E-14	MLE
	1	0.632121	0.030271	3.62E-01	0.394508	0.058926	0.633248	1.02E-03	0.633248	1.01E-03	MLE
	1.5	0.179245	0.00605	3.00E-02	0.09533	7.28E-03	0.179948	1.77E-04	0.179948	1.76E-04	MLE
	2	0.060587	0.001923	3.44E-03	0.030797	9.11E-04	0.06077	2.43E-05	0.06077	2.42E-05	MLE
	2.5	0.025275	0.000787	6.00E-04	0.012729	1.62E-04	0.025384	4.21E-06	0.025384	4.20E-06	MLE

Table (15): Reliability estimator when ($\lambda = 2\ ,\ \theta = 1.5,\ c = 1$)

n	t_i	$Real$	$\hat{R}_{LS}$	$MSE(\hat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\hat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bayes})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	Best
30	0.5	0.999877	0.106974	0.79955	0.966255	2.62E-03	0.999734	2.49E-07	0.999717	2.78E-07	Bayes
	1	0.894601	0.028333	7.51E-01	0.63607	0.088195	0.896254	1.76E-03	0.894356	1.80E-03	Bayes
	1.5	0.632121	0.012506	3.84E-01	0.376482	8.43E-02	0.638787	4.15E-03	0.63577	4.12E-03	MLE
	2	0.430217	0.007151	1.79E-01	0.247794	5.00E-02	0.439324	3.67E-03	0.436653	3.61E-03	MLE
	2.5	0.302324	0.004573	8.87E-02	0.166311	2.81E-02	0.313733	2.52E-03	0.3116	2.46E-03	MLE
60	0.5	0.999877	0.099024	0.812697	0.975955	1.16E-03	0.99982	4.90E-08	0.999814	5.25E-08	MLE
	1	0.894601	0.024902	7.56E-01	0.65852	0.070488	0.895357	9.40E-04	0.894389	9.49E-04	Bayes
	1.5	0.632121	0.011352	3.85E-01	0.387738	7.46E-02	0.634883	2.33E-03	0.633364	2.32E-03	MLE
	2	0.430217	0.006459	1.80E-01	0.246909	4.28E-02	0.437454	1.83E-03	0.436113	1.80E-03	MLE
	2.5	0.302324	0.0041	8.89E-02	0.167438	2.47E-02	0.306001	1.16E-03	0.304949	1.15E-03	MLE
90	0.5	0.999877	0.09524	0.819205	0.978835	7.89E-04	0.999833	2.61E-08	0.999829	2.74E-08	Bayes
	1	0.894601	0.024439	7.57E-01	0.658297	0.067238	0.894618	6.05E-04	0.893966	6.09E-04	Bayes
	1.5	0.632121	0.010799	3.86E-01	0.393111	6.77E-02	0.635907	1.55E-03	0.634891	1.54E-03	MLE
	2	0.430217	0.006268	1.80E-01	0.243442	4.10E-02	0.432569	1.16E-03	0.43168	1.15E-03	MLE
	2.5	0.302324	0.003959	8.90E-02	0.164501	2.16E-02	0.305174	6.89E-04	0.304474	6.83E-04	MLE
120	0.5	0.999877	0.093022	0.823183	0.980041	6.90E-04	0.999841	1.93E-08	0.999839	2.01E-08	Bayes
	1	0.894601	0.02369	7.59E-01	0.66737	0.061493	0.895143	4.77E-04	0.894655	4.79E-04	Bayes
	1.5	0.632121	0.0109	3.86E-01	0.385422	6.80E-02	0.632434	1.09E-03	0.631671	1.08E-03	MLE
	2	0.430217	0.006077	1.80E-01	0.243484	3.98E-02	0.433474	8.53E-04	0.432805	8.47E-04	MLE
	2.5	0.302324	0.003906	8.91E-02	0.164959	2.22E-02	0.303867	5.00E-04	0.303343	4.97E-04	MLE
150	0.5	0.999877	0.090527	0.827515	0.982205	5.30E-04	0.99985	1.29E-08	0.999848	1.34E-08	Bayes
	1	0.894601	0.023862	7.58E-01	0.660535	0.062177	0.894207	3.97E-04	0.893814	3.99E-04	Bayes
	1.5	0.632121	0.010431	3.87E-01	0.395235	6.39E-02	0.633851	9.13E-04	0.633239	9.12E-04	MLE
	2	0.430217	0.005913	1.80E-01	0.247746	3.78E-02	0.432282	7.08E-04	0.431748	7.05E-04	MLE
	2.5	0.302324	0.003867	8.91E-02	0.163863	2.18E-02	0.30367	4.45E-04	0.303251	4.43E-04	MLE

<u>Table (16):</u> Reliability estimator when $(\lambda = 4\ ,\ \ \theta = 1.5\ ,\ \ c = 1)$

n	t_i	$Real$	$\hat{R}_{LS}$	$MSE(\hat{R}_{LS})$	$\hat{R}_{LM}$	$MSE(\hat{R}_{LM})$	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bayes})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	$Best$
30	0.5	1	0.258415	0.550843	1	1.10E-18	1	0.00E+00	1	0.00E+00	Bayes
	1	0.99367	0.01853	9.51E-01	0.902607	0.012774	0.992236	4.52E-05	0.992381	4.35E-05	MLE
	1.5	0.632121	0.003701	3.95E-01	0.400364	6.52E-02	0.637574	4.78E-03	0.639066	4.80E-03	Bayes
	2	0.271237	0.001169	7.29E-02	0.154772	1.72E-02	0.280014	2.15E-03	0.280986	2.18E-03	Bayes
	2.5	0.121553	0.000483	1.47E-02	0.065382	3.78E-03	0.124492	5.29E-04	0.124972	5.36E-04	Bayes
60	0.5	1	0.243978	0.571992	1	1.93E-22	1	0.00E+00	1	0.00E+00	Bayes
	1	0.99367	0.017436	9.53E-01	0.910478	0.009371	0.992721	2.30E-05	0.992792	2.25E-05	MLE
	1.5	0.632121	0.003448	3.95E-01	0.397936	6.05E-02	0.636898	2.29E-03	0.637655	2.30E-03	Bayes
	2	0.271237	0.001094	7.30E-02	0.149405	1.63E-02	0.275024	9.92E-04	0.275506	9.99E-04	Bayes
	2.5	0.121553	0.000448	1.47E-02	0.064474	3.55E-03	0.123041	2.27E-04	0.123279	2.28E-04	Bayes
90	0.5	1	0.239059	0.579311	1	4.51E-24	1	0.00E+00	1	0.00E+00	Bayes
	1	0.99367	0.016931	9.54E-01	0.911626	0.00838	0.993117	1.36E-05	0.993163	1.35E-05	MLE
	1.5	0.632121	0.003363	3.95E-01	0.397866	5.90E-02	0.634376	1.51E-03	0.634883	1.51E-03	Bayes
	2	0.271237	0.001065	7.30E-02	0.149455	1.56E-02	0.273962	6.03E-04	0.274284	6.06E-04	Bayes
	2.5	0.121553	0.000437	1.47E-02	0.063641	3.51E-03	0.122698	1.46E-04	0.122857	1.47E-04	Bayes
120	0.5	1	0.23606	0.583805	1	5.24E-25	1	0.00E+00	1	0.00E+00	Bayes
	1	0.99367	0.016666	9.55E-01	0.915379	0.007321	0.993307	9.21E-06	0.993341	9.11E-06	MLE
	1.5	0.632121	0.003319	3.95E-01	0.394854	5.92E-02	0.634312	1.20E-03	0.634692	1.20E-03	Bayes
	2	0.271237	0.001048	7.30E-02	0.147985	1.59E-02	0.272184	4.66E-04	0.272424	4.67E-04	Bayes
	2.5	0.121553	0.00043	1.47E-02	0.063234	3.52E-03	0.122481	1.08E-04	0.1226	1.08E-04	Bayes
150	0.5	1	0.2337	0.587385	1	2.94E-23	1	0.00E+00	1	0.00E+00	Bayes
	1	0.99367	0.016497	9.55E-01	0.917001	0.006825	0.993434	6.58E-06	0.993461	6.52E-06	MLE
	1.5	0.632121	0.003271	3.95E-01	0.398861	5.68E-02	0.63344	9.07E-04	0.633745	9.07E-04	Bayes
	2	0.271237	0.001042	7.30E-02	0.147334	1.58E-02	0.271978	3.55E-04	0.27217	3.55E-04	Bayes
	2.5	0.121553	0.000425	1.47E-02	0.063133	3.50E-03	0.122636	8.59E-05	0.122732	8.63E-05	Bayes

Conclusion

1. The best estimator for scale parameter is found by maximum likelihood first as explained by table (1)(2)(3)(4).and the second method is Bayes one ,and the third one is Least square method ,this is due to the important properties of ML Estimator

2. for certain chosen set values $(\lambda = 4, \theta = 0.5, c = 2)$, we find [MSE($\hat{\theta}$)] using lest square is the smallest.

3. The best estimator of reliability function is also by maximum likelihood method since$\hat{\theta}$ML is the best estimator ,so this results Reflect on R̂ML.

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