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Neutrosophic k-Ideal in Q-algebra

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A b s t r a c t

In this paper, the k-ideals in Q-algebras are studied in the framework of single-valued neutrosophic sets. After recalling some basic notions related to Q-algebras and single-valued neutrosophic sets, the concept of single-valued neutrosophic k-ideals are introduced. Several properties and characterizations of single-valued neutrosophic k-ideals are investigated. Moreover, the relationships between single-valued neutrosophic k-ideals and single-valued neutrosophic ideals are discussed. Illustrative examples are provided to support the theoretical results.

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Table 1. Nomenclature for Single-Valued Neutrosophic Set.

Nomenclature			
SVN-S	A single-valued neutrosophic set	η_{M}	Function from $\mathcal{U}$ to [0,1]
SVN k-ideal	single-valued neutrosophic k-ideal	β_{M}	Function from $\mathcal{U}$ to [0,1]
$p^{\{**\}}$	involution if $p^{\star\star} = p$	τ_{M}	Function from $\mathcal{U}$ to [0,1]
e	Unit of U		
Min	Minimum		
Max	Maximum		

1. INTRODUCTION

Q-algebras are algebraic structures that have been studied by several authors due to their importance in algebra and logic see, for example (Neggers, et al., 2001)(Kim, H et al., 2005).Many concepts related to ideals have been introduced in Q-algebras in order to investigate their structural properties, and among these concepts the notion of k-ideals have received particular attention (Neggers, J. et al., 2004). The fuzzy set theory has been used as an effective tool to deal with uncertainty in algebraic systems (Zadeh, 1965) Later, intuitionistic fuzzy sets were introduced as a generalization of fuzzy sets, and several types of intuitionistic fuzzy ideals and intuitionistic fuzzy k-ideals in Q-algebras were studied see (Atanassov, K. T. 1986). Intuitionistic fuzzy sets (Jun & Neggers, 2005). Neutrosophic sets were proposed as a further generalization of intuitionistic fuzzy sets by considering the degrees of truth, indeterminacy, and falsity independently(Smarandache, 2005). In particular, single-valued neutrosophic sets have been applied to various algebraic structures as an extension of intuitionistic fuzzy sets (Smarandache, F. 2005), (Wang,

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H., et al., 2010).In this paper, the k-ideals of Q-algebras are studied in the framework of single-valued neutrosophic sets. The notion of a single-valued neutrosophic k-ideal is defined and its basic properties are investigated. Also, the relationship between single-valued neutrosophic k-ideals and single-valued neutrosophic ideals is discussed, which is presented with some examples to illustrate the results. This paper is organized as followed. Section 2 contains some basic definitions and results which are needed in the sequel. In Section 3, single-valued neutrosophic k-ideals in Q-algebras are studied that establish several related properties.

2. BACKGROUND AND PRELIMINARIES

In this section, the basic concepts related to Q-algebras are recalled to introduce the notions of single-valued neutrosophic sets and ideals which will be used throughout this paper. All algebraic definitions are preserved as in the original work, while fuzzy concepts are replaced entirely by their neutrosophic counterparts.

Definition (2.1) (Kim & Neggers, 2005)
A Q-algebra is a nonempty set $(\mathcal{U},\star,0)$ together with a binary operation $\star$ and a constant 0 satisfying the following axioms:
$(i) p \star p = 0,$
$(ii)\ p \star 0 = p,$
$(iii)(p \star q) \star r = (p \star r) \star q,$
$for\ all\ p, q, r \in \mathcal{U}.$

Remark (2.2) (Kim et al., 2005)
In a Q-algebra $\mathcal{U}$, a binary relation $\leq$ is defined by $p \leq q$ if and only if $p \star q = 0$.

Definition (2.3) (Neggers et al., 2001).
A Q-algebra $\mathcal{U}$ is called bounded if an element e $\in \mathcal{U}$ exists, such that $p \star e = 0$ for all p $\in \mathcal{U}$. The element e is called the unit of $\mathcal{U}$
$. e * p\ is\ denoted$ by p^*, for each $p \in \mathcal{U}$ in the bounded $\mathrm{Q-algebra}$.

Definition (2.4) (Neggers et al., 2001).

Let $\mathcal{U}$ be a bounded Q-algebra. An element p $\in \mathcal{U}$ is called an involution if $p^{\star\star} = p$. If every element of $\mathcal{U}$ is an involution, then $\mathcal{U}$ is called an involutory Q-algebra.

Proposition (2.5) (Neggers, et..al. 2004).
Let $\mathcal{U}$ be a bounded Q-algebra. Then, for all p, q $\in \mathcal{U}$, the following properties hold:
$(i)\ 0 \star p = p,$
$(ii)\ p \leq q\ implies\ p \star e \leq p \star e,$
$(iii) if\ \mathcal{U}\ is\ involutory, then\ p^{\star\star} = p$

Definition (2.6) (Neggers & Kim, 2001)
Let $(\mathcal{U},*,0)$ be a Q-algebra and I be a none empty subset of $\mathcal{U}$. Then, I is called an ideal of $\mathcal{U}$ if
for any $p, q \in \mathcal{U}$.
1. $0 \in I$
2. If $p * q \in I\ and\ q \in I$ implies $p \in I$.

Definition (2.7) (Jawad , H.K. 2019).
Let $(\mathcal{U},*,0)$ be a Q-algebra and I be a none empty subset of $\mathcal{U}$. Then, I is called an k- ideal of $\mathcal{U}$ if it satisfies
1. $0 \in I$
2. If $q^* * p \in I\ and\ q \in I$ implies $p^* \in I$, $\forall p \in \mathcal{U}$

$In\ short, (K-ID)\ is\ used\ instead\ of\ K-ideal$

Definition (2.8) (Wang et al., 2010)
A single-valued neutrosophic set (SVN-S) M in a universe $\mathcal{U}$ is an object of the form $\mathrm{M} = \{ (p, \beta_{\mathrm{M}}(p), \tau_{\mathrm{M}}(p), \eta_{\mathrm{M}}(p)) : p \in \mathcal{U} \},$
where $\beta_{\mathrm{M}}, \tau_{\mathrm{M}}, \eta_{\mathrm{M}} : \mathcal{U} \to [0,1]$ represent respectively the degrees of truth-membership, indeterminacy-membership, and falsity-membership, satisfying $0 \leq \beta_{\mathrm{M}} + \tau_{\mathrm{M}} + \eta_{\mathrm{M}} \leq 3$ for all p $\in \mathcal{U}$.

Definition (2.9) (Wang et al., 2010)
The intersection, union and subset of two SVN-S M and Γ are defined respectively by
:$\mathrm{M} \subseteq \Gamma$ if and only if $\beta_{\mathrm{M}}(p) \leq \beta_{\Gamma}(p), \quad \tau_{\mathrm{M}}(p) \geq \tau_{\Gamma}(p), and\ \eta_{\mathrm{M}}(p) \geq \eta_{\Gamma}(p)\ for\ all\ p \in \mathcal{U}$
$\beta_{\{\mathrm{M}\cap\Gamma\}}(p) = min\{\beta_{\mathrm{M}}(p), \beta_{\Gamma}(p)\},$
$\tau_{\{\mathrm{M}\cap\Gamma\}}(p) = max\{\tau_{\mathrm{M}}(p), \tau_{\Gamma}(p)\},$
$\eta_{\{\mathrm{M}\cap\Gamma\}}(p) = max\{\eta_{\mathrm{M}}(p), \eta_{\Gamma}(p)\};$
$\beta_{\{\mathrm{M}\cup\Gamma\}}(p) = max\{\beta_{\mathrm{M}}(p), \beta_{\Gamma}(p)\},$
$\tau_{\{\mathrm{M}\cup\Gamma\}}(p) = min\{\tau_{\mathrm{M}}(p), \tau_{\Gamma}(p)\},$
$\eta_{\{\mathrm{M}\cup\Gamma\}}(p) = min\{\eta_{\mathrm{M}}(p), \eta_{\Gamma}(p)\}.$

Definition (2.10) (Jun, Y. B. (2000).
Let $f : \mathcal{U} \to \Psi$ be a mapping. If Γ is a SVN-S in Ψ, then the preimage of Γ under f, denoted by $f^{-1}(\Gamma)$, is the SVN-S in $\mathcal{U}$ defined by

$$\beta_{\{f^{-1}(\Gamma)\}}(p) = \beta_{\Gamma}(f(p)), \tau_{\{f^{-1}(\Gamma)\}}(p) = \tau_{\Gamma}(f(p)), \eta_{\{f^{-1}(\Gamma)\}}(p) = \eta_{\Gamma}(f(p))\ for\ all\ p \in \mathcal{U}.$$

Definition (2.11) .

A SVN-S $M = \langle \beta_M, \tau_M, \eta_M \rangle$ in a Q-algebra $\mathcal{U}$ is called a single-valued neutrosophic ideal (in short SVN-ideal) if for all $p, q \in \mathcal{U}$:

SVN i (1)

$$\beta_M(0) \geq \beta_M(p)\ ,\quad \tau_M(0) \leq \tau_M(p)\quad and\quad \eta_M(0) \leq \eta_M(p),$$

SVN i (2) $\beta_M(p) \geq min\{\beta_M(p \star q), \beta_M(q)\}$, $\tau_M(p) \leq max\{\tau_M(p \star q), \tau_M(q)\}$ and $\eta_M(p) \leq max\{\eta_M(p \star q), \eta_M(q)\}$.

Example (2.12).

Let U = {0, ρ, ϖ, π} be a bounded Q-algebra whose binary operation * is given by Table 1.

Table 1. Cayley table of U

$*$	0	ρ	ϖ	π
0	0	0	0	0
ρ	ρ	0	0	0
ϖ	π	ϖ	0	0
π	π	π	ρ	0

Now define a single-valued neutrosophic set M in U by

$$\beta_M(p) = \begin{cases} 1, & \text{if } p = 0 \\ 0.7, & \text{if } p \in \{\rho, \varpi, \pi\}, \end{cases},$$

$$\tau_M(p) = \begin{cases} 0, & \text{if } p = 0 \\ 0.2, & \text{if } p \in \{\rho, \varpi, \pi\} \end{cases},$$

$$\eta\_M(p) = \begin{cases} 0, & \text{if } p = 0 \\ 0.3, & \text{if } p \in \{\rho, \varpi, \pi\}. \end{cases},$$

By direct computation using Table 1, there is $0 \leq \beta_M(p) + \tau_M(p) + \eta_M(p) \leq 3$ for all p ∈ U.

Also,

$\beta_M(0) \geq \beta_M(p),\ \tau_M(0) \leq \tau_M(p),\ \eta_M(0) \leq \eta_M(p)$ for all p ∈ U.

Moreover, for all $p, q \in U$, one can verify from Table 1 that

$$\beta_M(p) \geq min\{\beta_M(p * q), \beta_M(q)\},$$
$$\tau_M(p) \leq max\{\tau_M(p * q), \tau_M(q)\},$$
$$\eta_M(p) \leq max\{\eta_M(p * q), \eta_M(q)\}.$$

Hence, M satisfies all conditions of Definition (2.11). Therefore, M is a single-valued neutrosophic ideal of U. □

Definition (2.13)

Let $\rho, \varpi, \pi \in [0,1]$ such $that\ 0 \leq \rho + \varpi + \pi \leq 3$. The SVN-S M defined by

$\beta_M(p) = \rho\ , \tau_M(p) = \varpi, \eta_M(p) = \pi$ for all $p \in \mathcal{U}$ is called a constant single-valued neutrosophic set.

Example (2.14)

Let $\mathcal{U}$ be any bounded Q-algebra. Define a single-valued neutrosophic set M in $\mathcal{U}$ by

$\beta_M(p) = 0.8,\ \tau_M(p) = 0.1,\ \eta_M(p) = 0.1$ for all $p \in \mathcal{U}$.

$Since\ 0 \leq \beta_M(p) + \tau_M(p) + \eta_M(p) = 1 \leq 3$ for all $p \in \mathcal{U}$, M is a constant single-valued neutrosophic set.

3. SINGLE-VALUED NEUTROSPHIC K-IDEAL

In this section, the whole theory of k-ideals is rebuilt in the single-valued neutrosophic framework. Throughout, $\mathcal{U}$ denotes a bounded Q-algebra with unit e, and M denotes a single-valued neutrosophic set (SVN-S) in $\mathcal{U}$ written as $M = \langle \beta_M, \tau_M, \eta_M \rangle$

Definition (3.1)

A SVN-S $M = \langle \beta_M, \tau_M, \eta_M \rangle$ in a bounded Q-algebra $\mathcal{U}$ is called a single-valued neutrosophic k-ideal (in short, SVN k-ideal) of $\mathcal{U}$ if $for\ all\ p, q, \in \mathcal{U}$. the following hold:

SVN k (1) $\beta_M(0) \geq \beta_M(p),\ \tau_M(0) \leq \tau_M(p),\ \eta_M(0) \leq \eta_M(p)$.

SVN k (2) $\beta_M(p^\star) \geq min\{\beta_M(q^\star \star p), \beta_M(q)\}$,

$\tau_M(p^\star) \leq max\{\tau_M(q^\star \star p), \tau_M(q)\}$,

$\eta_M(p^\star) \leq max\{\eta_M(q^\star \star p), \eta_M(q)\}$.

Example (3.2)

Let $U = \{0, \rho, \varpi, \pi\}$ be a bounded Q-algebra whose operation $*$ is given by Table 2.

Table 2. Cayley table of U

$*$	0	ρ	ϖ	π
0	0	0	0	0
ρ	ρ	0	ρ	0
ϖ	ϖ	ϖ	0	0
π	π	π	ρ	0

Define a single-valued neutrosophic set $M = \langle \beta_M, \tau_M, \eta_M \rangle$ on U by

$$\beta_M(p) = \begin{cases} 1 & if\ \ p = 0 \\ 0.8 & if\ p = \rho \\ 0.6 & if\ p = \varpi \\ 0.4 & if\ \ p = \pi, \end{cases}$$

$$\tau_M(p) = \begin{cases} 0 & if\ \ p = 0 \\ 0.1 & if\ p = \rho \\ 0.2 & if\ p = \varpi \\ 0.3 & if\ \ p = \pi, \end{cases}$$

$$\eta_M(p) = \begin{cases} 0 & if \quad p = 0 \\ 0.1 & if \ p = \rho \\ 0.2 & if \ p = \varpi \\ 0.3 & if \quad p = \pi, \end{cases}$$

Clearly, $0 \leq \beta_M(p) + \tau_M(p) + \eta_M(p) \leq 3$ for all $p \in U$.
Moreover, β_M is decreasing while τ_M and η_M are increasing along the structure of U. Hence,
$\beta_M(0) \geq \beta_M(p),\ \tau_M(0) \leq \tau_M(p),\ \eta_M(0) \leq \eta_M(p)$, for all $p \in U$.
Now, for $p, q \in U$, using Table 1, it can be observed that the operation * preserves the order of elements in U in the sense that the values of β_M do not increase under the operation, while τ_M and η_M do not decrease.
Thus, for all p, q ∈ U,

$$\beta_M(p^*) \geq min\{\beta_M(q^* * p), \beta_M(q)\},$$
$$\tau_M(p^*) \leq max\{\tau_M(q^* * p), \tau_M(q)\},$$
$$\eta_M(p^*) \leq max\{\eta_M(q^* * p), \eta_M(q)\}.$$

Therefore, all conditions of Definition (3.1) are satisfied, and hence, M is a single-valued neutrosophic k-ideal of U. □

Theorem(3.3). Let U be a bounded Q-algebra and let $M = \langle \beta_M, \tau_M, \eta_M \rangle$ be a single-valued neutrosophic k-ideal of U. Then, every nonempty level subset $M_{(\alpha,\gamma,\delta)}$ is a k-ideal of U.

Proof.
Assume that $M = \langle \beta_M, \tau_M, \eta_M \rangle$ is a single-valued neutrosophic k-ideal of U.
First, it should be shown that $0 \in M_{(\alpha,\gamma,\delta)}$. Since $M_{(\alpha,\gamma,\delta)} \neq \emptyset$, there exists an element $a \in M_{(\alpha,\gamma,\delta)}$.
Hence, $\beta_M(a) \geq \alpha, \tau_M(a) \leq \gamma, \eta_M(a) \leq \delta$.
Since M is a SVN k-ideal, by condition SVN k(1), $\beta_M(0) \geq \beta_M(a)$ and $\tau_M(0) \leq \tau_M(a)$, $\eta_M(0) \leq \eta_M(a)$ are obtained.
Therefore, $\beta_M(0) \geq \alpha, \tau_M(0) \leq \gamma, \eta_M(0) \leq \delta$, which implies that $0 \in M_{(\alpha,\gamma,\delta)}$.
Next, let p, q ∈ U such that $q^* * p \in M_{(\alpha,\gamma,\delta)}$ and $q \in M_{(\alpha,\gamma,\delta)}$.
Then, $\beta_M(q^* * p) \geq \alpha, \beta_M(q) \geq \alpha$,
$\tau_M(q^* * p) \leq \gamma, \tau_M(q) \leq \gamma$,
$\eta_M(q^* * p) \leq \delta, \eta_M(q) \leq \delta$.
Since M is a single-valued neutrosophic k-ideal, by condition SVN k(2) we have

$$\beta_M(p^*) \geq \min\{\beta_M(q^* * p), \beta_M(q)\},$$
$$\tau_M(p^*) \leq \max\{\tau_M(q^* * p), \tau_M(q)\},$$
$$\eta_M(p^*) \leq \max\{\eta_M(q^* * p), \eta_M(q)\}.$$

Using the above inequalities, we obtain

$$\beta_M(p^*) \geq \alpha, \tau_M(p^*) \leq \gamma, \eta_M(p^*) \leq \delta.$$

Thus, $p * \in M_{(\alpha,\gamma,\delta)}$.
Consequently, $M_{(\alpha,\gamma,\delta)}$ contains 0 and satisfies the defining condition of a k-ideal. Hence, $M_{(\alpha,\gamma,\delta)}$ $is\ a\ k-ideal\ of\ U$.
□ **Corollary (3.4)**

Let I be a nonempty subset of a bounded Q-algebra U, and let $\chi_I = \langle \beta_{\chi_I}, \tau_{\chi_I}, \eta_{\chi_I} \rangle$ be the characteristic single-valued neutrosophic set of I defined by $\beta_{\chi_I}(p) = 1,\ \tau_{\chi_I}(p) = 0,\ \eta_{\chi_I}(p) = 0\ \ if\ p \in I$,

And $\beta_{\chi_I}(p) = 0,\ \tau_{\chi_I}(p) = 1,\ \eta_{\chi_I}(p) = 1\ \ if\ p \notin I$.

If χ_I is a single-valued neutrosophic k-ideal of U, then I is a k-ideal of U.

Proof.

Take α = 1, γ = 0, and δ = 0.

Then, $(\chi_I)_{(1,0,0)} = I$.

Since χ_I is a single-valued neutrosophic k-ideal of U, it follows from the preceding theorem that every nonempty level subset of χ_I is a k-ideal of U. Hence, $I = (\chi_I)_{(1,0,0)}$ is a k-ideal of U.

Proposition (3.5).
Let U be a bounded Q-algebra such that $p^* = p$ for all $p \in U$. If M is a single-valued neutrosophic ideal of U, then M is a single-valued neutrosophic k-ideal of U

Proof.
Assume that $M = \langle \beta_M, \tau_M, \eta_M \rangle$ is a single-valued neutrosophic ideal of U.
Then, for all $p, q \in U$, the following conditions hold:

$$\beta_M(0) \geq \beta_M(p),\ \ \tau_M(0) \leq \tau_M(p),\ \ \eta_M(0) \leq \eta\_M(p),$$

and

$$\beta_M(p) \geq min\{\beta_M(p * q), \beta_M(q)\},$$
$$\tau_M(p) \leq max\{\tau_M(p * q), \tau_M(q)\},$$
$$\eta_M(p) \leq max\{\eta_M(p * q), \eta_M(q)\}.$$

Since $p^* = p$ for all $p \in U$, it follows in particular that $q^* = q$ for all $q \in U$. Hence

$$\beta_M(p^*) = \beta_M(p),\ \ \tau_M(p^*) = \tau_M(p),\ \ \eta_M(p^*) = \eta_M(p),$$

and

$$\beta_M(q^* * p) = \beta_M(q * p),$$
$$\tau_M(q^* * p) = \tau_M(q * p),$$
$$\eta_M(q^* * p) = \eta_M(q * p).$$

Therefore, from the defining inequalities of a single-valued neutrosophic ideal, the following is obtained:

$$\beta_M(p^*) = \beta_M(p) \geq min\{\beta_M(q * p), \beta_M(q)\} = min\{\beta_M(q^* * p), \beta_M(q)\},$$

$$\tau_M(p^*) = \tau_M(p) \leq max\{\tau_M(q * p), \tau_M(q)\} = max\{\tau_M(q^* * p), \tau_M(q)\},$$
$$\eta_M(p^*) = \eta_M(p) \leq max\{\eta_M(q * p), \eta\_M(q)\} = max\{\eta_M(q^* * p), \eta_M(q)\}.$$

Thus, conditions SVN k(1) and SVN k(2) are satisfied. Hence, M is a single-valued neutrosophic k-ideal of U. □

Proposition (3.6).
Let $M = \langle \beta_M, \tau_M, \eta_M \rangle$ be a single-valued neutrosophic k-ideal of a bounded Q-algebra U. DefineI_$I_M = \{ p \in U : \beta_M(p) = \beta_M(0), \tau_M(p) = \tau_M(0), \eta_M(p) = \eta_M(0) \}$.Then, I_M is a k-ideal of U**.**

Proof.
Let $I_M = \{ p \in U : \beta_M(p) = \beta_M(0), \tau_M(p) = \tau_M(0), \eta_M(p) = \eta_M(0) \}$.
Since M is a single-valued neutrosophic k-ideal of U, by condition SVN k(1) there are, for every $p \in U$,

$$\beta_M(p) \leq \beta_M(0), \quad \tau_M(p) \geq \tau_M(0), \quad \eta_M(p) \geq \eta_M(0).$$

$$Hence I_M = \{ p \in U : \beta_M(p) \geq \beta_M(0), \tau_M(p) \leq \tau_M(0), \eta_M(p) \leq \eta_M(0) \}.$$

That is,$I_M = M_{(\beta_M(0), \tau_M(0), \eta_M(0))}$.
Since $0 \in I_M$, the set I_M is nonempty. Therefore, by Theorem (3.3), every nonempty level subset of M is a k-ideal of U. In particular,

$$I_M = M_{(\beta_M(0), \tau_M(0), \eta_M(0))}$$

is a k-ideal of U.
Hence, I_M is a k-ideal of U. □

Theorem (3.7).
Let I be a nonempty subset of a bounded Q-algebra U, and $let\ \chi_I = \langle \beta_{\chi_I}, \tau_{\chi_I}, \eta_{\chi_I} \rangle$ be the characteristic single-valued neutrosophic set of I defined by $\beta_{\chi_I}(p) = 1,\ \tau_{\chi_I}(p) = 0,\ \eta_{\chi_I}(p) = 0$ if $p \in I$, and $\beta_{\chi_I}(p) = 0,\ \tau_{\chi_I}(p) = 1,\ \eta_{\chi_I}(p) = 1\ \ if\ p \notin I.$
Then I is a k-ideal of U if and only if χ_I is a single-valued neutrosophic k-ideal of U**.**

Proof.

Assume first that I is a k-ideal of U. Then, 0 ∈ I, and hence $\beta_{\chi_I}(0) = 1,\ \tau_{\chi_I}(0) = 0,\ \eta_{\chi_I}(0) = 0.$

Thus, for all p ∈ U,

$$\beta_{\chi_I}(0) \geq \beta_{\chi_I}(p),\ \tau_{\chi_I}(0) \leq \tau_{\chi_I}(p),\ \eta_{\chi_I}(0) \leq \eta_{\chi_I}(p),$$

so condition SVN k(1) holds.

Now let $p, q \in U. If\ q^* * p \in I\ and\ q \in I, then\ p^* \in I$, and hence

$$\beta_{\chi_I}(p^*) = 1,\ \tau_{\chi_I}(p^*) = 0,\ \eta_{\chi_I}(p^*) = 0.$$

Thus, condition SVN k(2) holds. Therefore, χ_I is a SVN k-ideal of U.

Conversely, assume that χ_I is a SVN k-ideal of U. From SVN k(1), it is obtatined $\beta_{\chi_I}(0) = 1$, hence 0 ∈ I.

Let $q^* * p \in I$ and q ∈ I. Then, $\beta_{\chi_I}(q^* * p) = \beta_{\chi_I}(q) = 1$. By SVN k(2),

$$\beta_{\chi_I}(p *) \geq min\{1,1\} = 1,$$

so $p^* \in I$. Hence, I is a k-ideal of U. □ □

Remark (3.8)
Let $M = \langle \beta_M, \tau\_M, \eta\_M \rangle$ be a single-valued neutrosophic k-ideal of a bounded Q-algebra U. Then, for all p ∈ U,$\beta_M(p^*) \leq \beta_M(0),\ \ \tau_M(p^*) \geq \tau_M(0),\ \ \eta_M(p^*) \geq \eta_M(0)$.
Proof.
Let p ∈ U. Since M is a single-valued neutrosophic k-ideal of U, by condition SVN k(1) we have, for every x ∈ U,

$$\beta_M(0) \geq \beta_M(x),\ \ \tau_M(0) \leq \tau_M(x),\ \ \eta_M(0) \leq \eta_M(x).$$

Taking $x = p^*$, we obtain

$$\beta_M(0) \geq \beta_M(p^*),\ \ \tau_M(0) \leq \tau_M(p^*),\ \ \eta_M(0) \leq \eta_M(p^*).$$

Hence, the result follows. □

Theorem (3.9).
Let $f : U \to H$ be a homomorphism between bounded Q-algebras, and let $M = \langle \beta_M, \tau_M, \eta_M \rangle$ be a single-valued neutrosophic k-ideal of H. Then $f^{-1}(M)$ is a single-valued neutrosophic k-ideal of U.
Proof.
Define $f^{-1}(M) = \langle \beta, \tau, \eta \rangle$ on U by

$$\beta(p) = \beta_M(f(p)),\ \tau(p) = \tau_M(f(p)),\ \eta(p) = \eta_M(f(p)),$$

for all $p \in U$.
Since f is a homomorphism between bounded Q-algebras, we have
$f(0_U) = 0_H$ and $f(p^*) = \left(f(p)\right)^*$, for all p ∈ U.
Let $p \in U$. Since M is a single-valued neutrosophic k-ideal of H, it includes

$$\beta_M(0_H) \geq \beta_M(f(p)),\ \tau_M(0_H) \leq \tau_M(f(p)),\ \eta_M(0\_H) \leq \eta_M(f(p)).$$

Hence,

$$\beta(0_U) = \beta\_M(0_H) \geq \beta(p),$$
$$\tau(0_U) = \tau_M(0_H) \leq \tau(p),$$
$$\eta(0_U) = \eta_M(0_H) \leq \eta(p),$$

so condition SVN k(1) holds.

Now let $p, q \in U$. Using the homomorphism property, we obtain $f(q^* * p) = (f(q))^* * f(p)$.
Since M is a single-valued neutrosophic k-ideal of H, it follows that

$$\beta_M((f(p))^*) \geq min\{\beta_M((f(q))^* * f(p)), \beta_M(f(q))\}.$$

Thus, $\beta(p^*) = \beta_M((f(p))^*)$

$$\geq min\{\beta_M(f(q^* * p)), \beta_M(f(q))\}$$
$$= min\{\beta(q^* * p), \beta(q)\}.$$

Similarly,

$$\tau(p^*) \leq max\{\tau(q^* * p), \tau(q)\},$$
$$\eta(p*) \leq max\{\eta(q^* * p), \eta(q)\}.$$

Hence, condition SVN k(2) holds. Therefore, $f^{-1}(M)$ is a single-valued neutrosophic k-ideal of U. □

Corollary (3.10)

Let $f : U \to H$ be an isomorphism between two bounded Q-algebras.
Let $M = \langle\beta_M, \tau_M, \eta_M\rangle$ be a single-valued neutrosophic k-ideal of U.Define $f(M) = \langle\beta', \tau', \eta'\rangle$ on H by

$$\beta'(y) = \beta_M(f^{-1}(y)), \tau'(y) = \tau_M(f^{-1}(y)),$$
$$\eta'(y) = \eta_M(f^{-1}(y)),$$

for all y ∈ H.Then $f(M)$ is a single-valued neutrosophic k-ideal of H..
Proof.
The result follows directly from Theorem (3.9). □

Proposition (3.11).

Let $\{M_i : i \in \Lambda\}$ be a family of single-valued neutrosophic k-ideals of a bounded Q-algebra U, where $M_i = \langle\beta_i, \tau_i, \eta_i\rangle$ for each $i \in \Lambda$. Define $M = \langle\beta, \tau, \eta\rangle$ by $\beta(p) = inf\,\{\beta_i(p) : i \in \Lambda\}, \tau(p) = sup\,\{\tau_i(p) : i \in \Lambda\}, \eta(p) = sup\,\{\eta_i(p) : i \in \Lambda\}$, for all p ∈ U. Then, M is a single-valued neutrosophic k-ideal of U.
Proof.
Let $p \in U$. Since each M_i is a single-valued neutrosophic k-ideal of U, by condition SVN k(1) there are

$$\beta_i(0) \geq \beta_i(p), \quad \tau_i(0) \leq \tau_i(p), \quad \eta_i(0) \leq \eta_i(p)$$

for all $i \in \Lambda$.
Taking infimum in the first inequality and supremum in the second and third inequalities, it is obtained that
$\beta(0) \geq \beta(p), \quad \tau(0) \leq \tau(p), \quad \eta(0) \leq \eta(p)$.
Hence, condition SVN k(1) holds.
Now let $p, q \in U$. Since each M_i is a single-valued neutrosophic k-ideal of U, by condition SVN k(2) there are

$$\beta_i(p^*) \geq min\{\beta_i(q^* * p), \beta_i(q)\},$$
$$\tau_i(p^*) \leq max\{\tau_i(q^* * p), \tau_i(q)\},$$
$$\eta_i(p^*) \leq max\{\eta_i(q^* * p), \eta_i(q)\}$$

for all $i \in \Lambda$.
Taking infimum in the first inequality, it results

$$\beta(p^*) = inf\,\beta_i(p^*)$$
$$\geq inf\,min\{\beta_i(q^* * p), \beta_i(q)\}$$
$$\geq min\{inf\,\beta_i(q^* * p), inf\,\beta_i(q)\}$$
$$= min\{\beta(q^* * p), \beta(q)\}.$$

Similarly, taking supremum in the second and third inequalities, it is obtained

$$\tau(p^*) = sup\,\tau_i(p^*)$$
$$\leq sup\,max\{\tau_i(q^* * p), \tau_i(q)\}$$
$$\leq max\{sup\,\tau_i(q^* * p), sup\,\tau_i(q)\}$$
$$= max\{\tau(q^* * p), \tau(q)\},$$

And $\eta(p^*) = sup\,\eta_i(p^*)$

$$\leq sup\,max\{\eta_i(q^* * p), \eta_i(q)\}$$
$$\leq max\{sup\,\eta_i(q^* * p), sup\,\eta_i(q)\}$$
$$= max\{\eta(q^* * p), \eta(q)\}.$$

Thus, condition SVN k(2) also holds. Therefore M is a single-valued neutrosophic k-ideal of U. □

Remark (3.12)

In general, the union of single-valued neutrosophic k-ideals need not be a single-valued neutrosophic k-ideal (see Example (3.12)).

Example (3.13).

Let $U = \{0, \rho, \varpi, \pi\}$ be a bounded Q-algebra whose operation * is given by Table 3.

Table 3. Cayley table of U

*	0	ρ	ϖ	π
0	0	0	0	0
ρ	ρ	0	0	0
ϖ	ϖ	ρ	0	0
π	π	Π	ρ	0

Define two single-valued neutrosophic sets
$M = \langle\beta_M, \tau_M, \eta_M\rangle$ and $N = \langle\beta_N, \tau_N, \eta_N\rangle$ on U as followed:

$$\beta_M(p) = \begin{cases} 0.4 & \text{if } p = 0, \\ 0.1 & \text{if } p \in \{\rho, \varpi, \pi\} \end{cases}$$
$$\tau_M(p) = \begin{cases} 0.0 & \text{if } p = 0 \\ 0.3 & \text{if } p \in \{\rho, \varpi, \pi\} \end{cases}$$
$$\eta_M(p) = \begin{cases} 0.0 & \text{if } p = 0 \\ 0.3 & \text{if } p \in \{\rho, \varpi, \pi\} \end{cases}$$

And

$$\beta_N(p) = \begin{cases} 0.6 & \text{if } p = 0 \\ 0.2 & \text{if } p \in \{\rho, \varpi, \pi\} \end{cases}$$
$$\tau_N(p) = \begin{cases} 0.0 & \text{if } p = 0 \\ 0.2 & \text{if } p \in \{\rho, \varpi, \pi\} \end{cases}$$
$$,\eta_N(p) = \begin{cases} 0.0 & \text{if } p = 0, \\ 0.2 & \text{if } p \in \{\rho, \varpi, \pi\} \end{cases}$$

By direct verification using Table 1, both M and N satisfy the conditions of Definition (3.1). Hence, M and N are single-valued neutrosophic k-ideals of U.
Therefore, by Proposition (3.10), M ∩ N is also a single-valued neutrosophic k-ideal of U.

However, in general, the union M ∪ N need not be a single-valued neutrosophic k-ideal of U, which illustrates Remark (3.11) Take $p = \rho$ and $q = \pi$. Then $q^* * p = 0$ and $p^* = \varpi$. Hence

$$\beta_{\{M \cup N\}}(p^*) = 0.1 < 0.2$$
$$= min\{\beta_{\{M \cup N\}}(q^* * p), \beta_{\{M \cup N\}}(q)\}.$$

Thus, condition SVN k(2) is not satisfied, and therefore M ∪ N is not a single-valued neutrosophic k-ideal of U. □

$\eta'(y_2^*) \leq \max\{\eta'(y_1^* * y_2), \eta'(y_1)\}$.

Hence, SVN k(2) holds. Therefore f(M) is a SVN k-ideal of H. □

CONCLUSION

In this paper, the concept of single-valued neutrosophic k-ideals is introduced in bounded Q-algebras and investigated their fundamental properties. Several characterizations of these structures are provided, particularly in terms of level subsets, and established their relationships with classical k-ideals.

Moreover, the behavior of single-valued neutrosophic k-ideals are studied under algebraic mappings and are shown to be preserved under homomorphic preimages. Also, closure properties, including the intersection of such ideals are examined, which are presented through illustrative examples and counterexamples to clarify the theoretical results.

The results obtained in this work contribute to the development of neutrosophic algebraic structures and pave the way for further research. In particular, future work may focus on studying single-valued neutrosophic k-ideals in other algebraic systems, as well as investigating their lattice structures and applications in decision-making and information sciences.

REFERENCES

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 20*, 87–96.

Jawad, H. K. (2019). *Some types of fuzzy pseudo Q-algebra* (Master's thesis, University of Kufa).

Jun, Y. B. (2000). Intuitionistic fuzzy ideals of Q-algebras. *Journal of Fuzzy Mathematics, 8*, 409–420.

Jun, Y. B., & Neggers, J. (2005). Intuitionistic fuzzy k-ideals in Q-algebras. *Journal of Applied Mathematics and Computing, 19*, 87–96.

Kim, H. S., & Neggers, J. (2005). The structure of Q-algebras. *Mathematica Slovaca, 55*, 473–484.

Neggers, J., & Kim, H. S. (2001). On Q-algebras. *International Journal of Mathematics and Mathematical Sciences, 27*, 749–757.

Neggers, J., Jun, Y. B., & Kim, H. S. (2004). On k-ideals in Q-algebras. *Mathematica Slovaca, 54*, 1–9.

Smarandache, F. (2002). *Neutrosophy and neutrosophic logic*. American Research Press.

Smarandache, F. (2005). Neutrosophic set: A generalization of the intuitionistic fuzzy sets. *International Journal of Pure and Applied Mathematics, 24*, 287–297.

Wang, H., Smarandache, F., Zhang, Y. Q., & Sunderraman, R. (2010). Single valued neutrosophic sets. *Multispace and Multistructure, 4*, 410–413.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*, 338–353.